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Jet Propagation and Mach Cone Formation in (3+1)dimensional Ideal Hydrodynamics

Barbara Betz

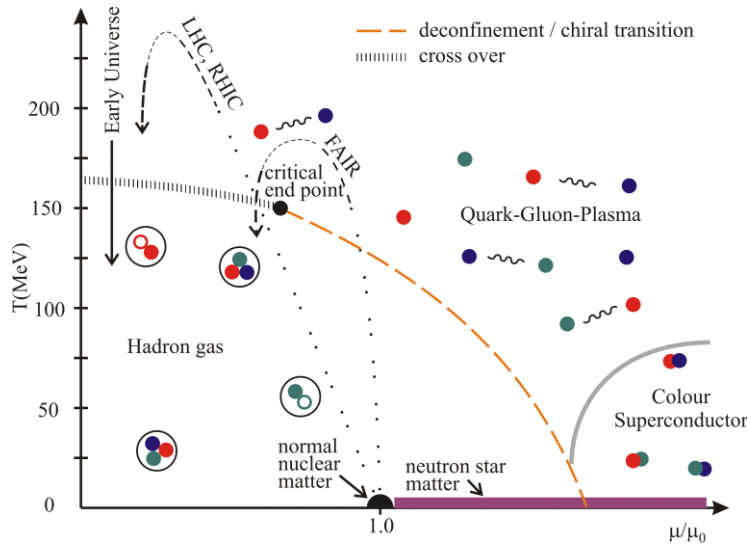
and

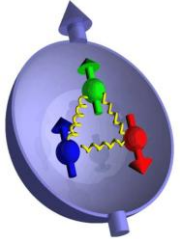
Miklos Gyulassy, Jorge Noronha, Dirk Rischke,
Giorgio Torrieri

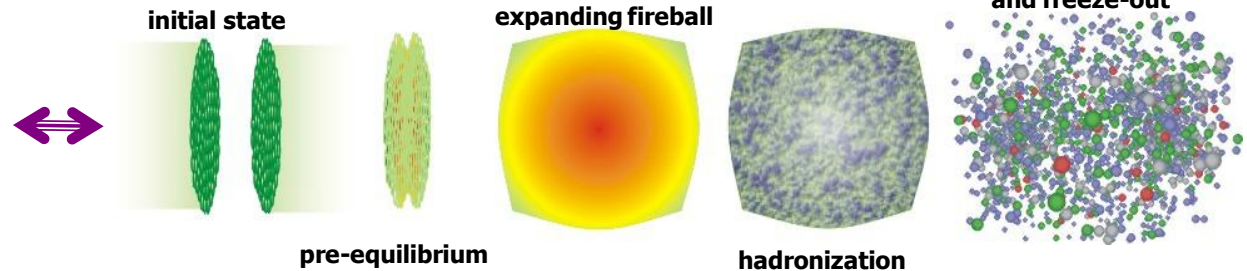
Phys. Rev. C **79**, 034902 (2009), Phys. Lett. B **675**, 340 (2009),
Nucl. Phys. A **830**, 777c (2009), arXiv:1005.5461

Explore The Matter in a HIC

- Why do we want to study jets & Mach cones?
- Why do we use hydrodynamics?

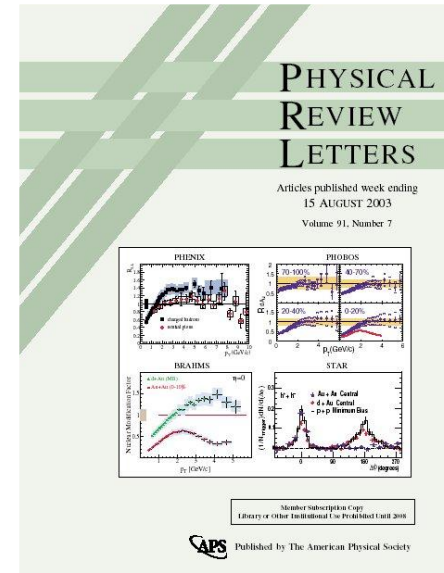


- Insights into theory of strong interactions (QCD) 
- Medium created in heavy-ion (HIC) collisions similar to the one created after Big Bang
- Explore the phase diagram of QCD with HIC



S. Bass, Talk Quark Matter 2001

RHIC Results



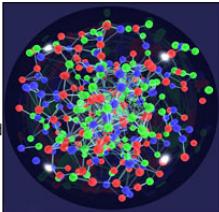
BBC NEWS LIVE BBC NEWS CHANNEL

Last Updated: Tuesday, 19 April, 2005, 16:26 GMT 17:26 UK

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Early Universe was 'liquid-like'

Physicists say they have created a new state of hot, dense matter by crashing together the nuclei of gold atoms.



The high-energy collisions prised open the nuclei to reveal their most basic particles, known as quarks and gluons.

The researchers, at the US Brookhaven National Laboratory, say these particles were seen to behave as an almost perfect "liquid".

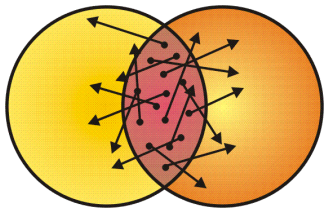
The impression is of matter that is more strongly interacting than predicted

- Jets are suppressed
- System is dense and opaque

- Medium behaves like a fluid

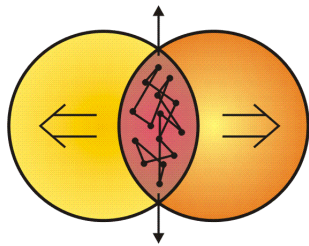
The Expanding Medium

From first principles, it is unclear if medium is ...



„dust“

Particles don't interact,
expansion independent
of initial shape

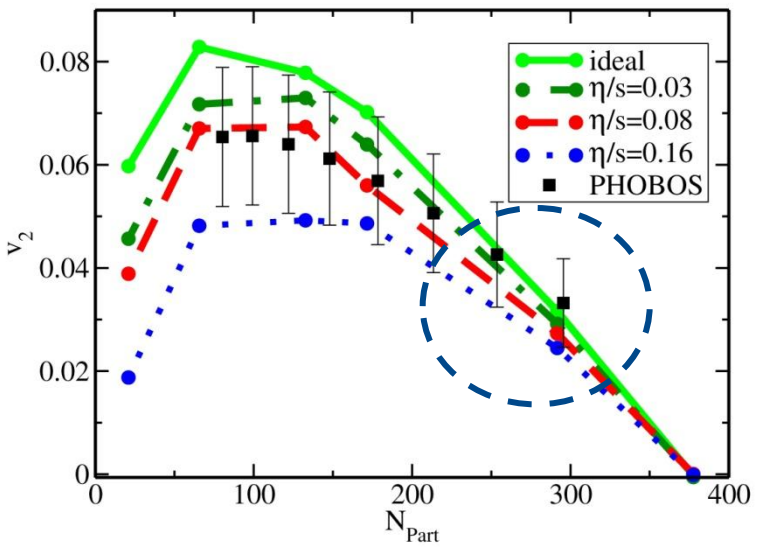


fluid

Particles interact,
expansion determined
by density gradient

Hydrodynamics: azimuthal anisotropy of emitted particles, parametrized by v_2

$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left[1 + 2 \sum_{n=1}^{\infty} v_n \cos(n\phi) \right]$$



- Data described by hydrodynamics
 - Small η/s
- Medium behaves like an almost ideal fluid

P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99**,172301 (2007)

Hydrodynamics

→ Medium created in a HIC can be described using hydrodynamics

- Hydrodynamics represents (local) conservation of

energy-momentum

$$\partial_\mu T^{\mu\nu} = 0, \quad \mu, \nu = 0, \dots, 3$$

(local) charge

$$\partial_\mu N_i^\mu = 0, \quad i = 1, \dots, n$$

- For **ideal** hydrodynamics in local thermodynamical equilibrium

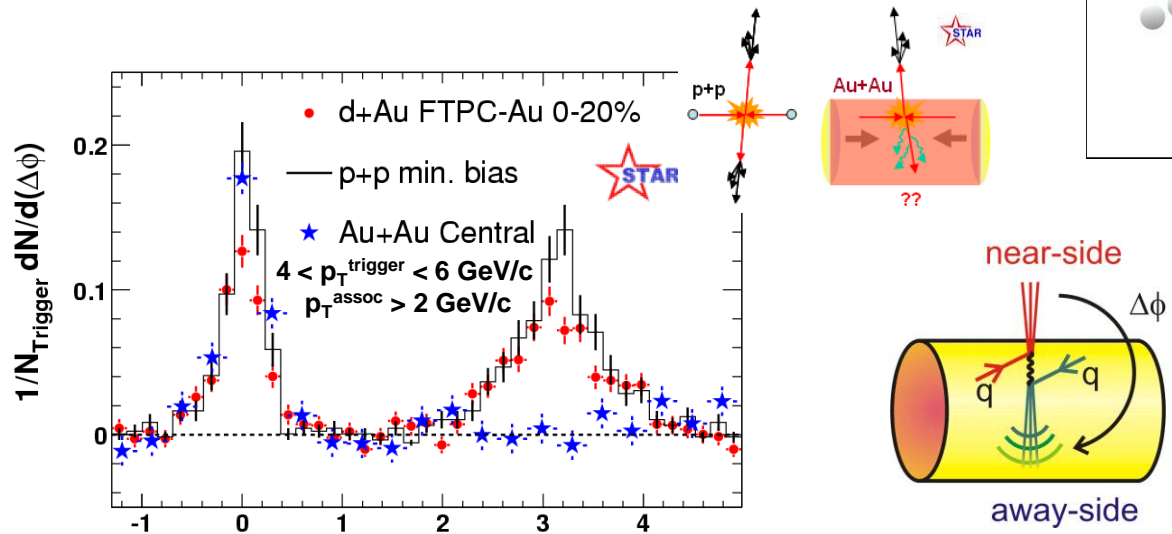
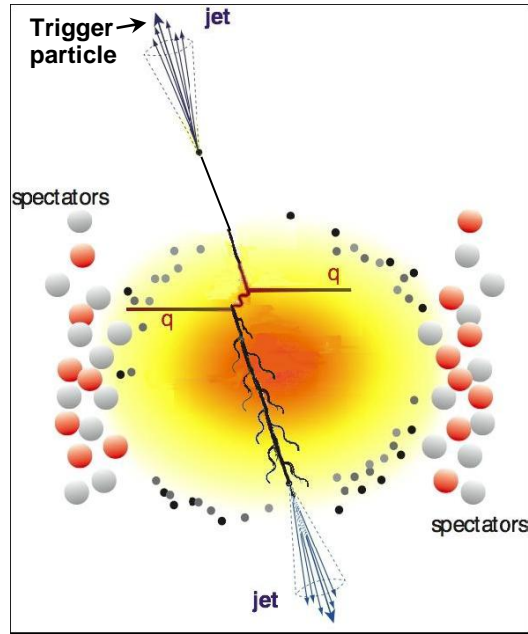
$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - p g^{\mu\nu}, \quad g^{\mu\nu} = \text{diag}(+, -, -, -)$$

$$N_i^\mu = n_i u^\mu, \quad u^\mu = \gamma(1, \vec{v}), \quad \gamma = (1 - v^2)^{-1/2}$$

- Equation of State** $p = p(\varepsilon, n_i)$
- Viscous** hydrodynamics will not be considered in our calculations

Jet - Studies in HIC I

- Jet moving through dense matter, depositing its energy
- should eventually disappear
- Jet suppression: signal for creation of opaque matter (Quark-Gluon Plasma)



STAR, Phys. Rev. Lett. **91** (2003) 072304 $\Delta \phi$ (radians)

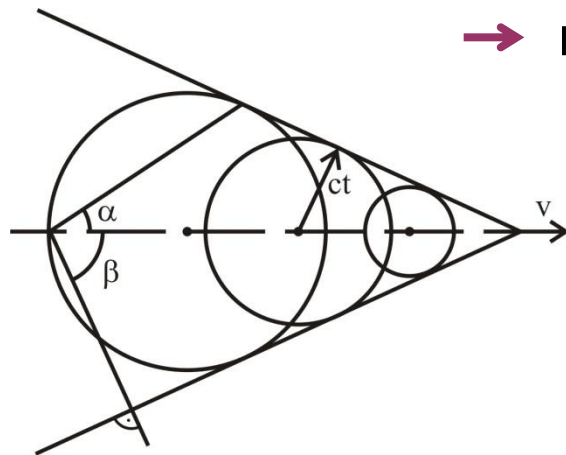
→ Can energy lost by jets tell us something about medium properties?

Jet - Studies in HIC II

Can energy lost by jets tell us something about medium properties?

→ Idea: Propagation of fast parton generates a Mach cone pattern

H. Stöcker, Nucl. Phys. A **750**, 121 (2005), J. Casalderrey-Solana et al. Nucl. Phys. A **774**, 577 (2006)



→ reflect interaction of jet with medium

Mach cone angle sensitive to EoS

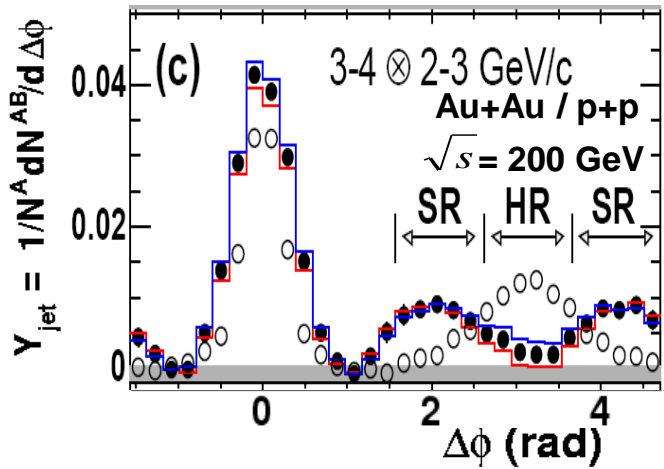
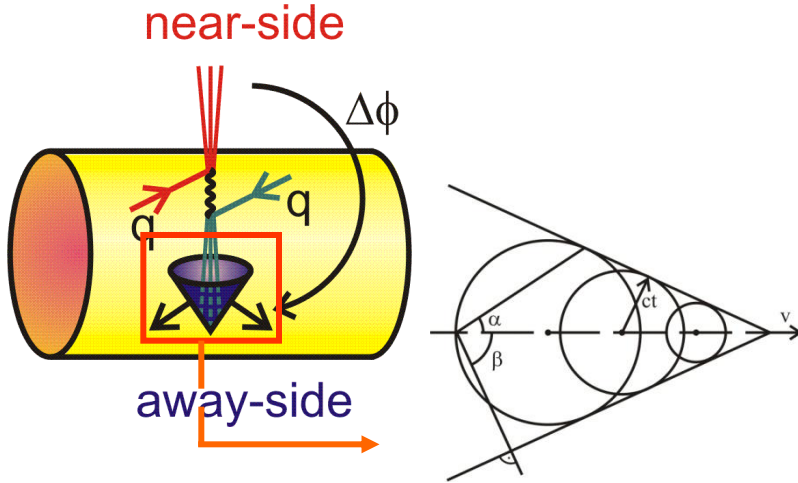
$$\cos \phi_M = \cos \beta = c_s / v_{\text{jet}}$$

By observation:

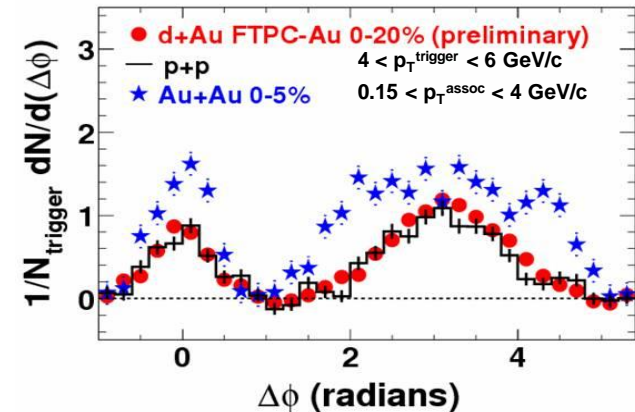
- Confirm fast thermalization
- Study EoS of the fluid

Jet - Studies in HIC III

- Generation of Mach cone pattern
- Redistribution of energy to lower p_T particles
- Re-appearance of the away-side for low and intermediate p_T^{assoc}



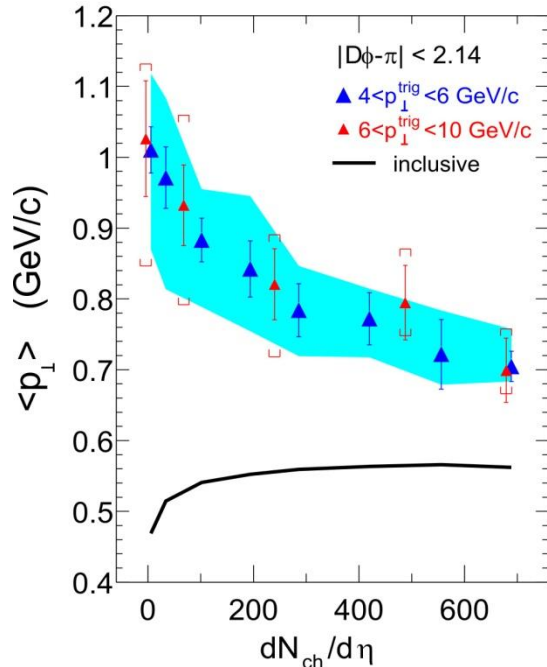
PHENIX, Phys. Rev. C 77, 011901 (2008)



STAR, Nucl. Phys. A 774, 129 (2006)

The Theory

Modelling of Jets



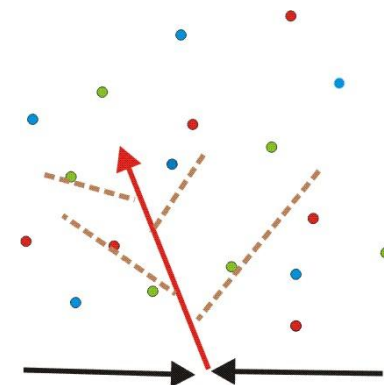
STAR, Phys. Rev. Lett. **95**, 152301 (2005)

Jets can be modelled using (ideal) hydrodynamics:

$$\partial_\mu T^{\mu\nu} = S^\nu$$

residue of energy and momentum given by the jet

- Different jet deposition mechanisms
 - GLV, BDMPS, ASW, AMY, Higher Twist



The Freeze-out Prescription

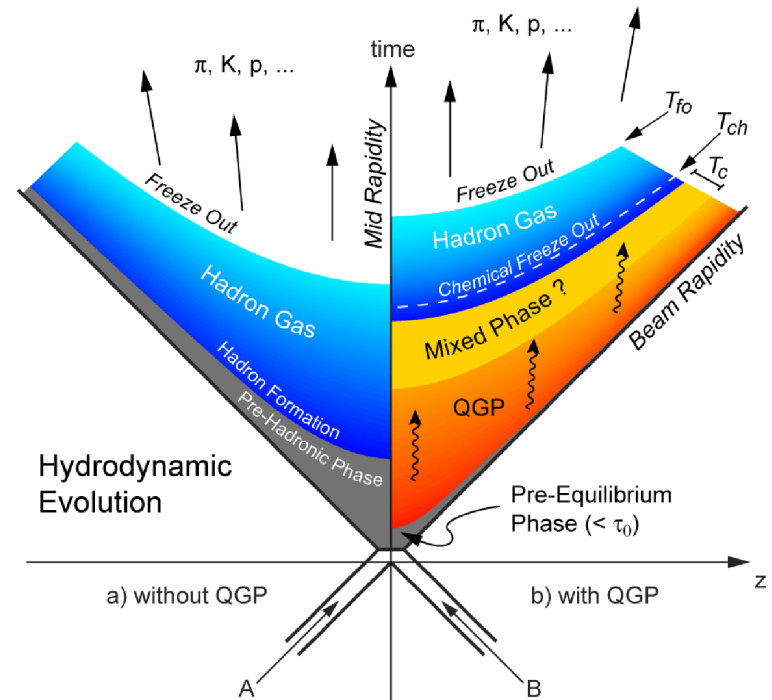
Cooper-Frye Freeze-out:

$$\frac{dN}{p_T dp_T d\phi} = C \int dV \exp[-\gamma(E - p_i v^i)/T]$$

↳ $e + \vec{p} \cdot \vec{v}$

→ mainly flow driven

- Assumption of isochronous/isothermal freeze-out
- No interaction afterwards



<http://www.rnc.blb.gov/ssalur/www/Research3.html>

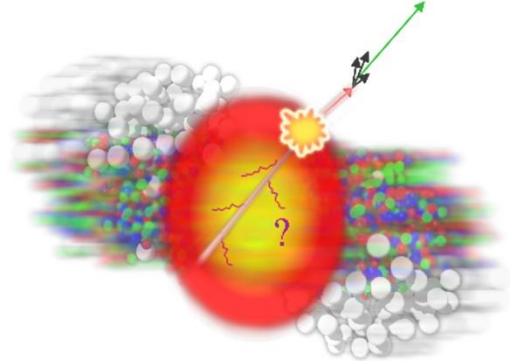
The Static Medium

Stopped Jet I

Applying a static medium and an ideal Gas EoS for massless gluons

↳ Maximal fluid response

Assume: Near-side jet is not modified by medium



$$S^\nu = \int_{\tau_i}^{\tau_f} \frac{dM^\nu}{d\tau} \delta^{(4)} \left[x^\mu - x_{\text{jet}}^\mu(\tau) \right] d\tau$$

$$dM^\nu/d\tau = (dE/d\tau, d\vec{M}/d\tau) \quad x_{\text{jet}}^\mu = x_0^\mu + u_{\text{jet}}^\mu \tau$$

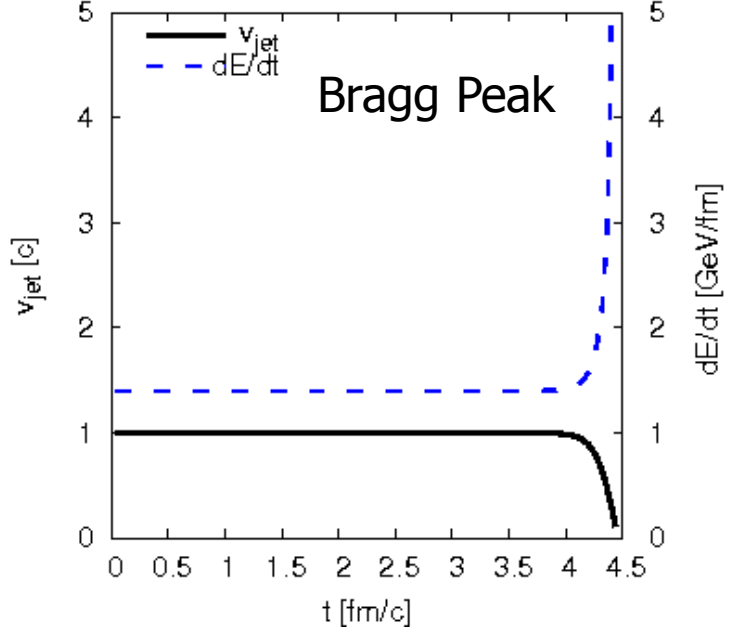
Jet decelerating from $v=0.999$ according to Bethe-Bloch formalism

$$\frac{dE}{dt}(t) = a \frac{1}{v_{\text{jet}}(t)} \quad a = -1.36 \text{ GeV/fm}$$

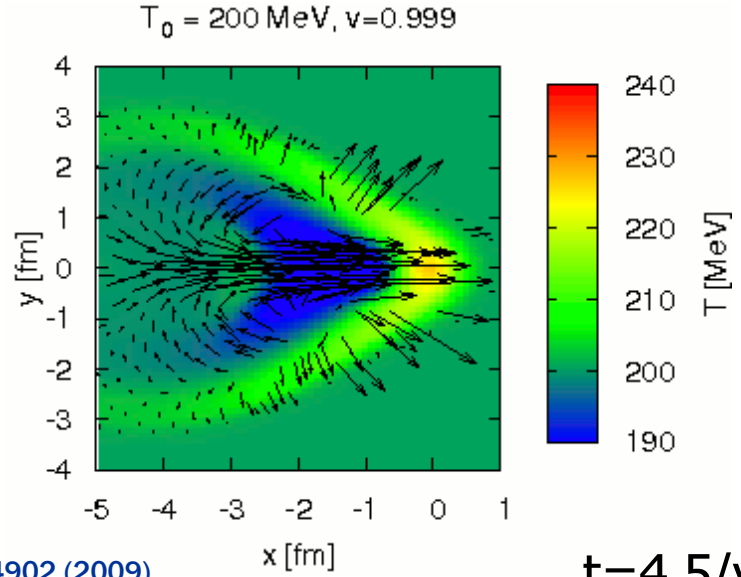
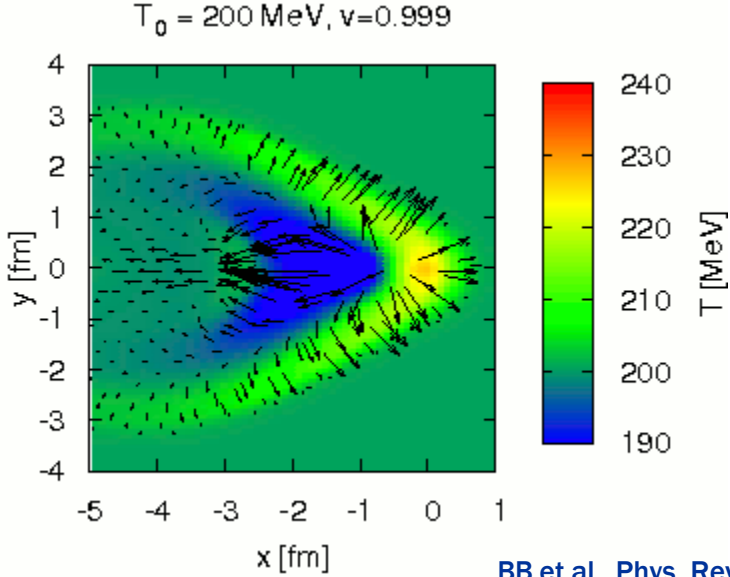
↑ adjusts path length

Simplest back-reaction from the medium

BB et al., Phys. Rev. C 79, 034902 (2009)

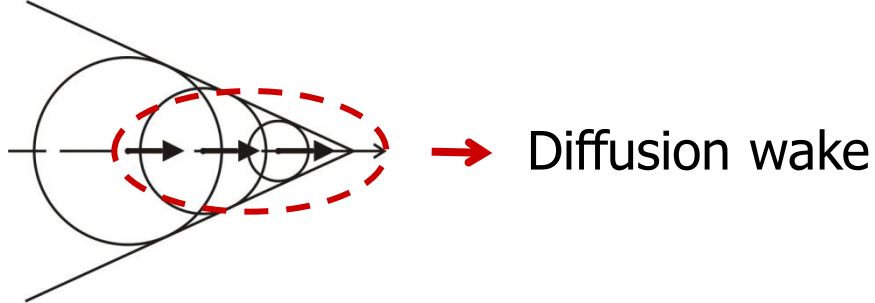
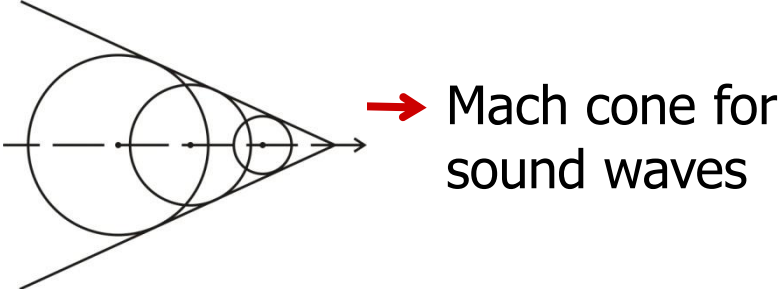


Stopped Jet II



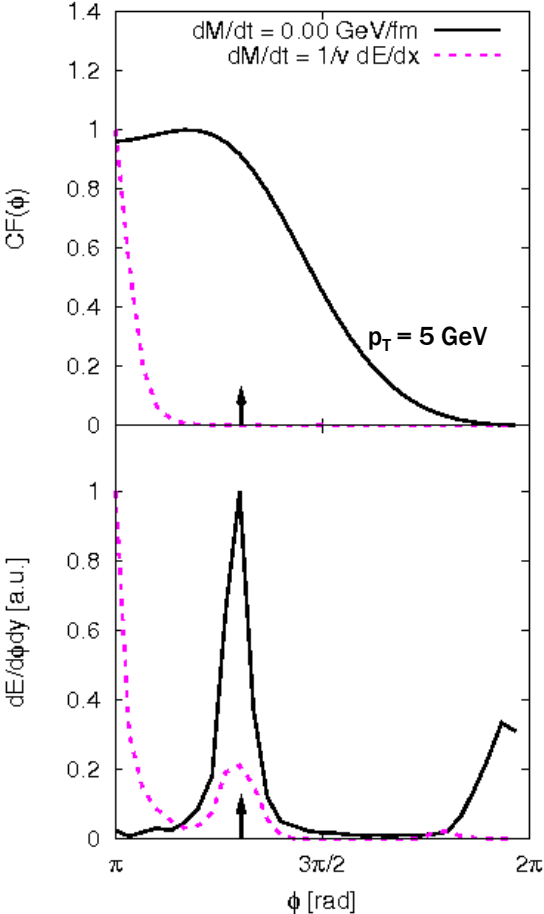
$$\frac{dE}{dt}(0) = 1.5 \frac{\text{GeV}}{\text{fm}} \quad \frac{dM}{dt}(0) = 0 \frac{\text{GeV}}{\text{fm}}$$

$$\frac{dE}{dt}(0) = v \frac{dM}{dt}(0) = 1.5 \frac{\text{GeV}}{\text{fm}}$$



Stopped Jet III

BB et al., Phys. Rev. C 79, 034902 (2009)



Diffusion wake causes peak in jet direction

Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

$$\frac{dN}{p_T dp_T dy d\phi} \Big|_{y=0} = \int_{\Sigma} d\Sigma_{\mu} p^{\mu} [f_{\text{Boltzmann}}(u \cdot p/T) - f_{\text{eq}}]$$

Energy Flow Distribution

$$\frac{dE}{d\phi dy} \Big|_{y=0} = \int d^3 \vec{x} E(\vec{x}) \delta[\phi - \Phi(\vec{x})]$$

$$\Phi(\vec{x}) = \tan^{-1} \left[\frac{M_y(\vec{x})}{M_x(\vec{x})} \right]$$

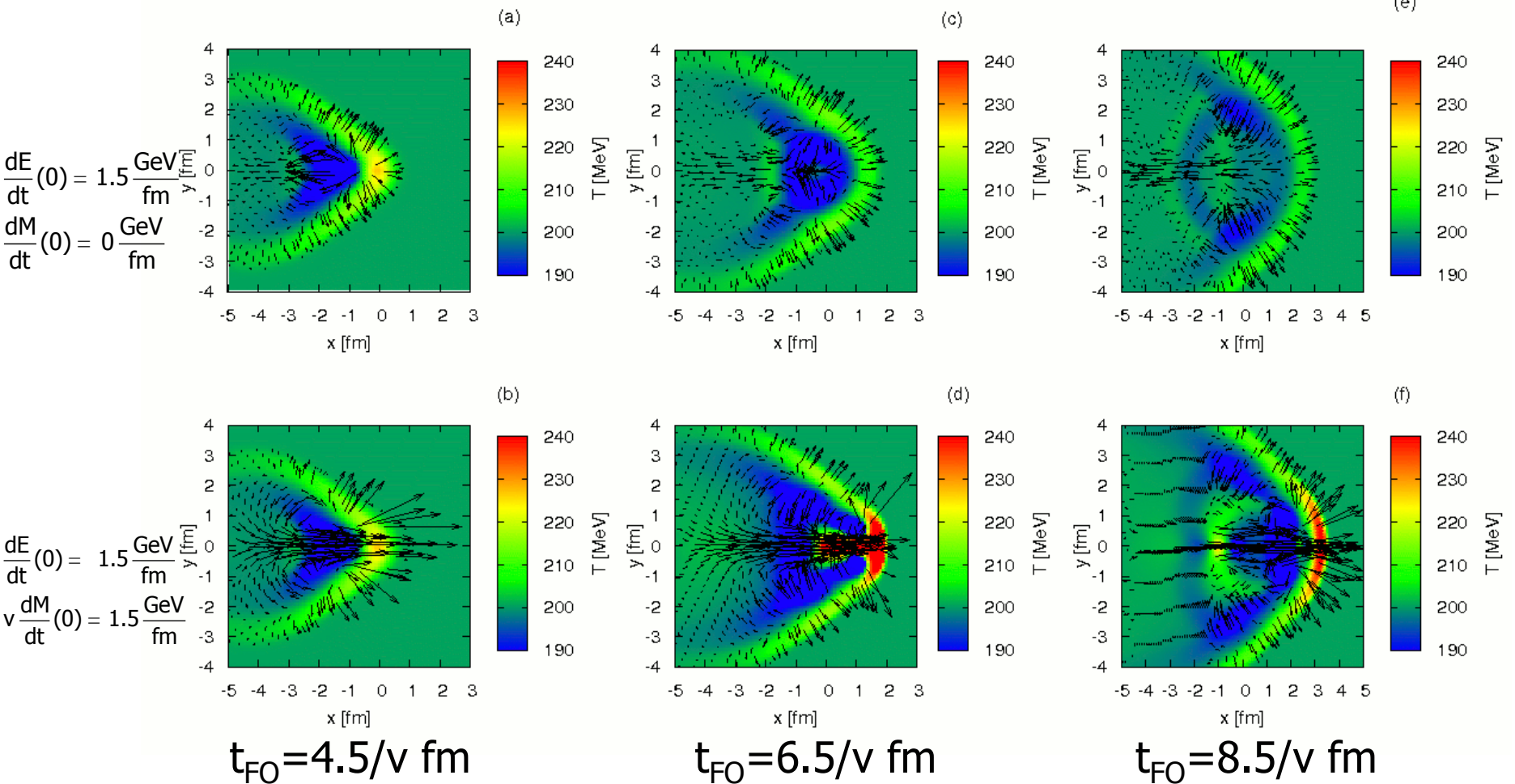
Assuming: Particles in subvolume will be emitted into the same direction

Stopped Jet IV

- Jet stops after $t=4.5/v$ fm

$T_0 = 200$ MeV, $v_0=0.999$

BB et al., Phys. Rev. C 79, 034902 (2009)



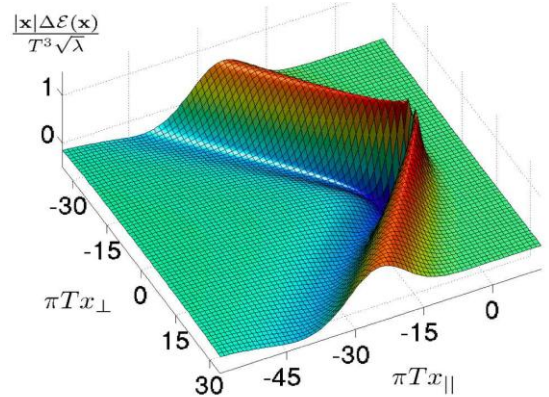
Diffusion wake still present → Vorticity conservation

Different Jet-Energy Loss Models

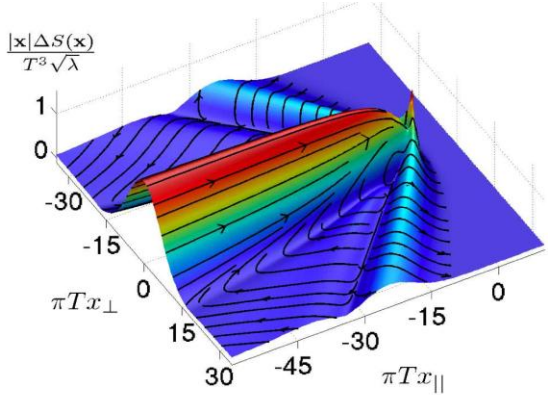
Modelling Jets using ...

Strongly-coupled theory
AdS/CFT

Energy density perturbation



Pointing vector perturbation



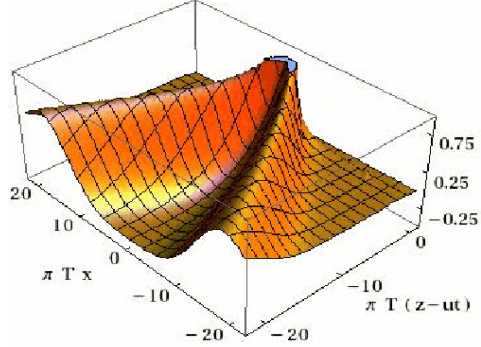
$v=0.75$

P. Chesler and L. Yaffe, *Phys. Rev. D* **78**, 045013 (2008)

Weakly-coupled theory
pQCD

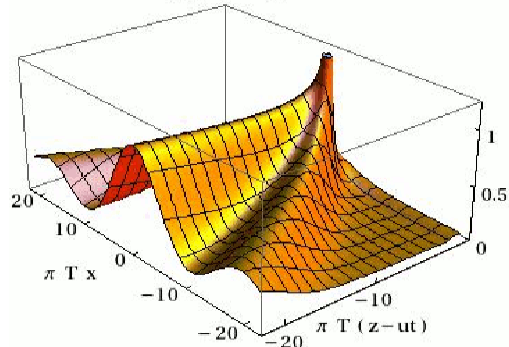
Energy density perturbation

$$\frac{|\vec{x}| \epsilon(\vec{x})}{m_D^2 T} \frac{\eta}{s} = 0.13 \quad (a)$$



Momentum density perturbation

$$\frac{|\vec{x}| g(\vec{x})}{m_D^2 T} \frac{\eta}{s} = 0.13 \quad (b)$$

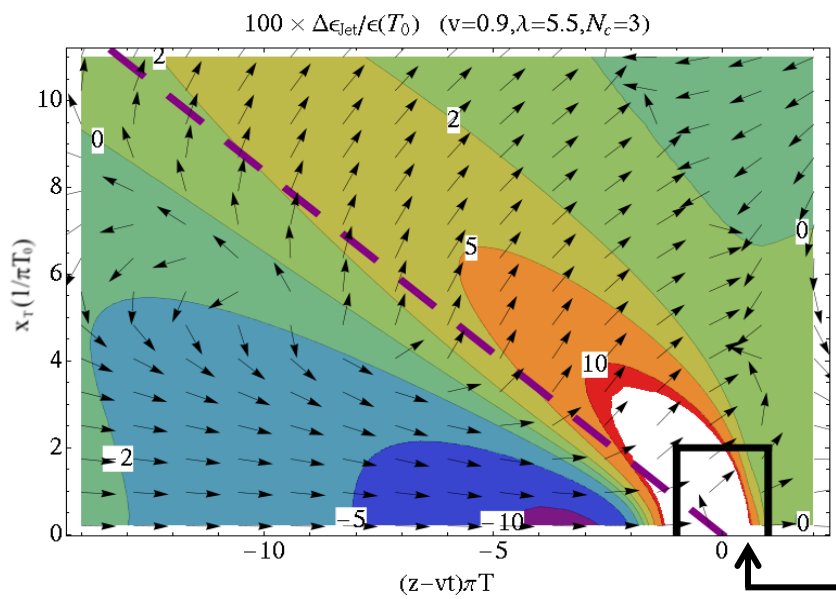


$v=0.99955$

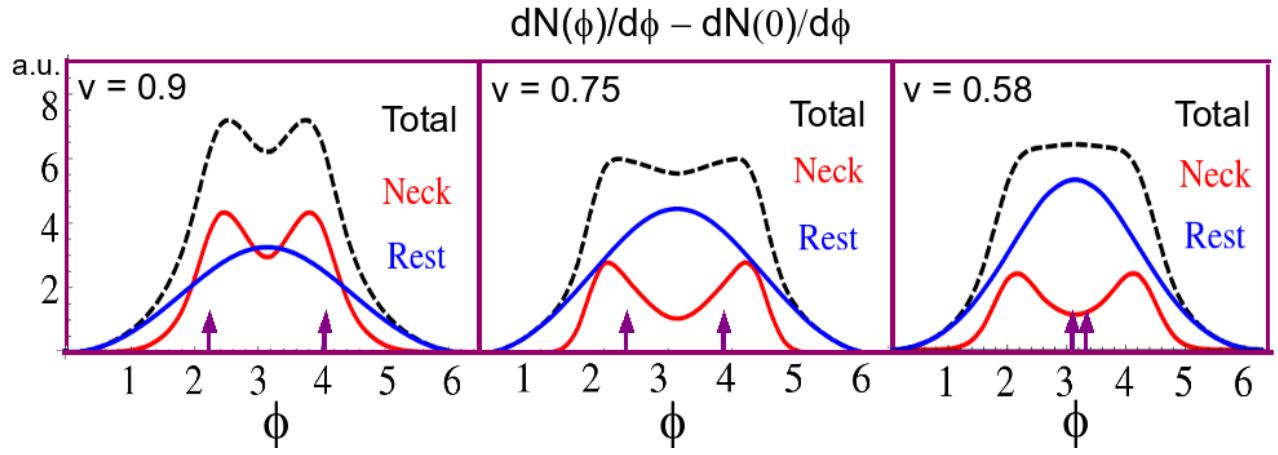
R. Neufeld et al, *Phys. Rev. C* **78**, 041901 (2008)

→ Conclusion about Mach cones?

Jets in AdS/CFT



J. Noronha et al., Phys. Rev. Lett. **102**, 102301 (2009)



$$\cos \phi_M = c_s / v_{\text{jet}}$$

Heavy Quark Jets

Heavy Quark Jets

Compare weakly and strongly coupled models using heavy punch-through jet
 Static medium and isochronous freeze-out needed for comparison

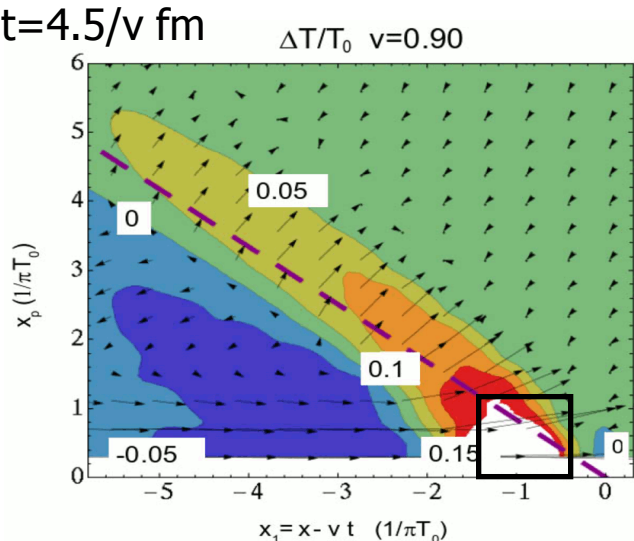
pQCD: Neufeld et al. source for a heavy quark

R. Neufeld et al, Phys. Rev. C 78, 041901 (2008)

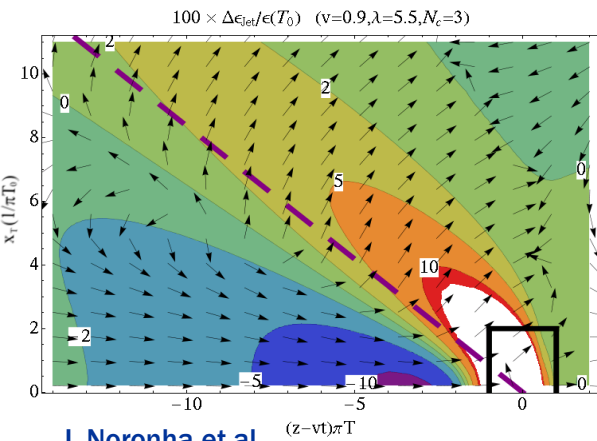
AdS/CFT: Stress tables with $\eta/s=1/(4\pi)$

S. Gubser et al, Phys. Rev. Lett. 100, 012301 (2008)

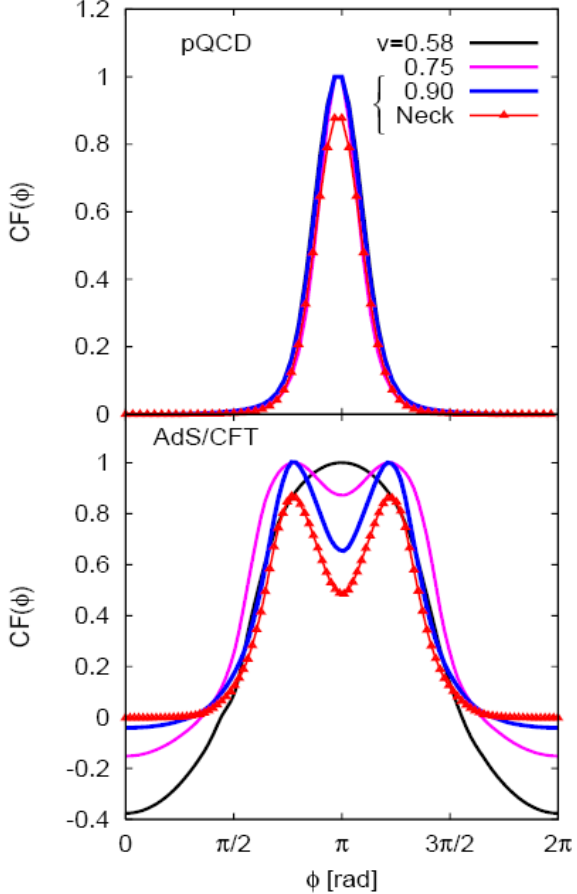
BB et al., Phys. Lett. B 675, 340 (2009)



BB et al., Phys. Lett. B 675, 340 (2009)



J. Noronha et al., Phys. Rev. Lett. 102, 102301 (2009)



$p_T = 3.14 \text{ GeV}$

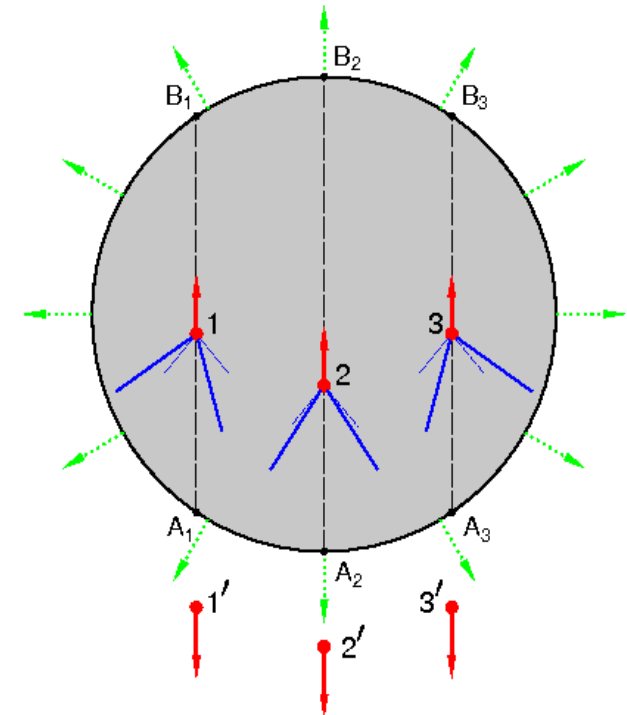
→ No Mach-like peaks: $\cos \phi_M = c_s / v_{jet}$

AdS/CFT: Strong influence of the Neck region

The Expanding Medium

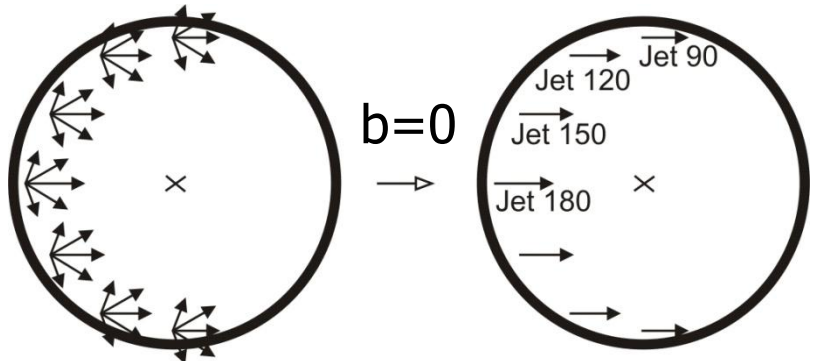
Expanding Medium I

- Consequences of expansion?
 - Radial flow, Elliptic flow
- Predictions:
 - Transverse flow causes distortion
 - Expansion broadens Mach cone angle
- Mach cones are sensitive to the background flow
- Qualitative, model-independent effect



Satarov et al, PLB 627:64 (2005)

Expanding Medium II



Experimental results based on many events
Consider different jet paths

A. K. Chaudhuri, Phys. Rev. C 75, 057902 (2007),
A. K. Chaudhuri, Phys. Rev. C 77, 027901 (2008)

- Apply Glauber initial conditions and an ideal Gas EoS for massless gluons
- Focus on radial flow contribution

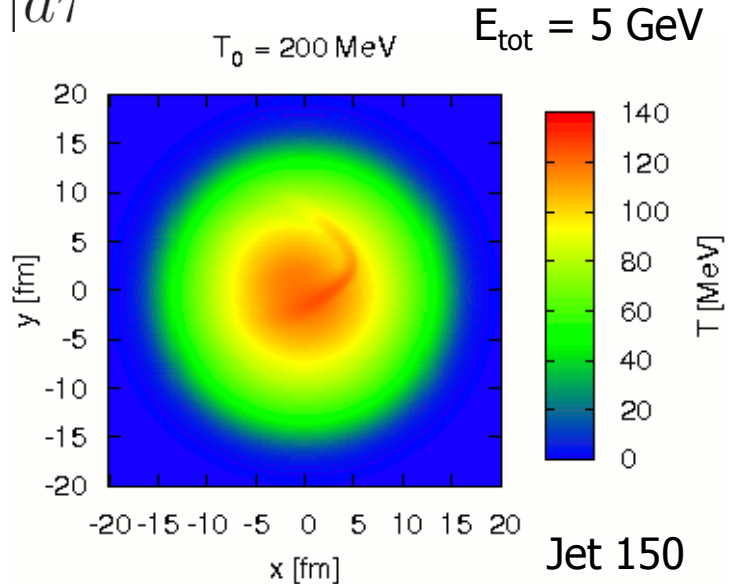
$$S^\nu = \int_{\tau_i}^{\tau_f} \frac{dM^\nu}{d\tau} \Big|_o \left[\frac{T(\tau)}{T_{\max}} \right]^3 \delta^{(4)} [x^\mu - x_{\text{jet}}^\mu(\tau)] d\tau$$

- Two-particle correlation
($T_{\text{freeze-out}} < T_{\text{crit}} = 130 \text{ MeV}$):

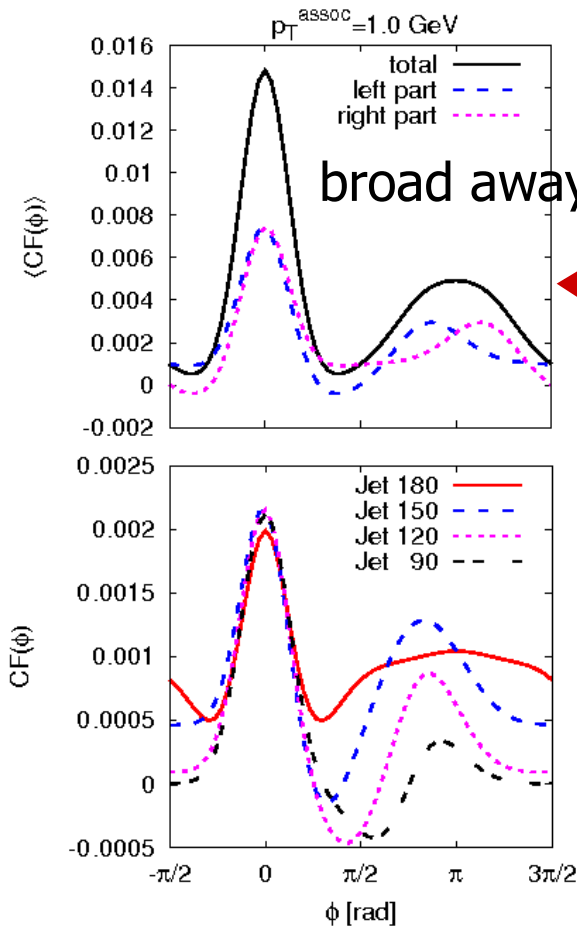
$$N(\phi) = A(\phi) + \int_0^{2\pi} d\phi^* N(\phi - \phi^*) f(\phi^*)$$

$$\langle CF(\phi) \rangle = \frac{\langle N \rangle(\phi) - \langle N_{\text{back}} \rangle(\phi)}{\int_0^{2\pi} \langle N_{\text{back}} \rangle(\phi) d\phi}$$

near-side jet

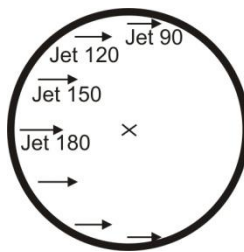


Expanding Medium III

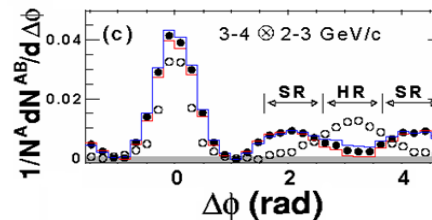
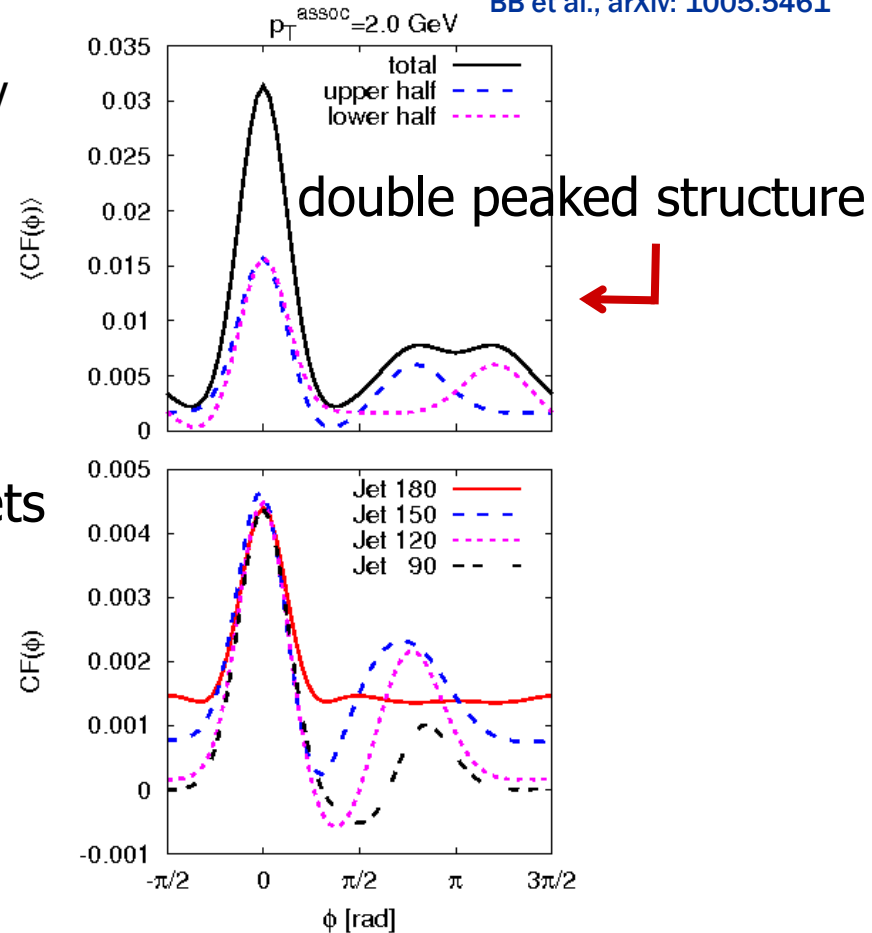
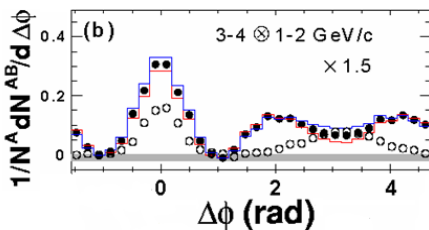


$E_{\text{tot}} = 5 \text{ GeV}$
 $p_T^{\text{trig}} = 3.5 \text{ GeV}$

due to
non-central jets



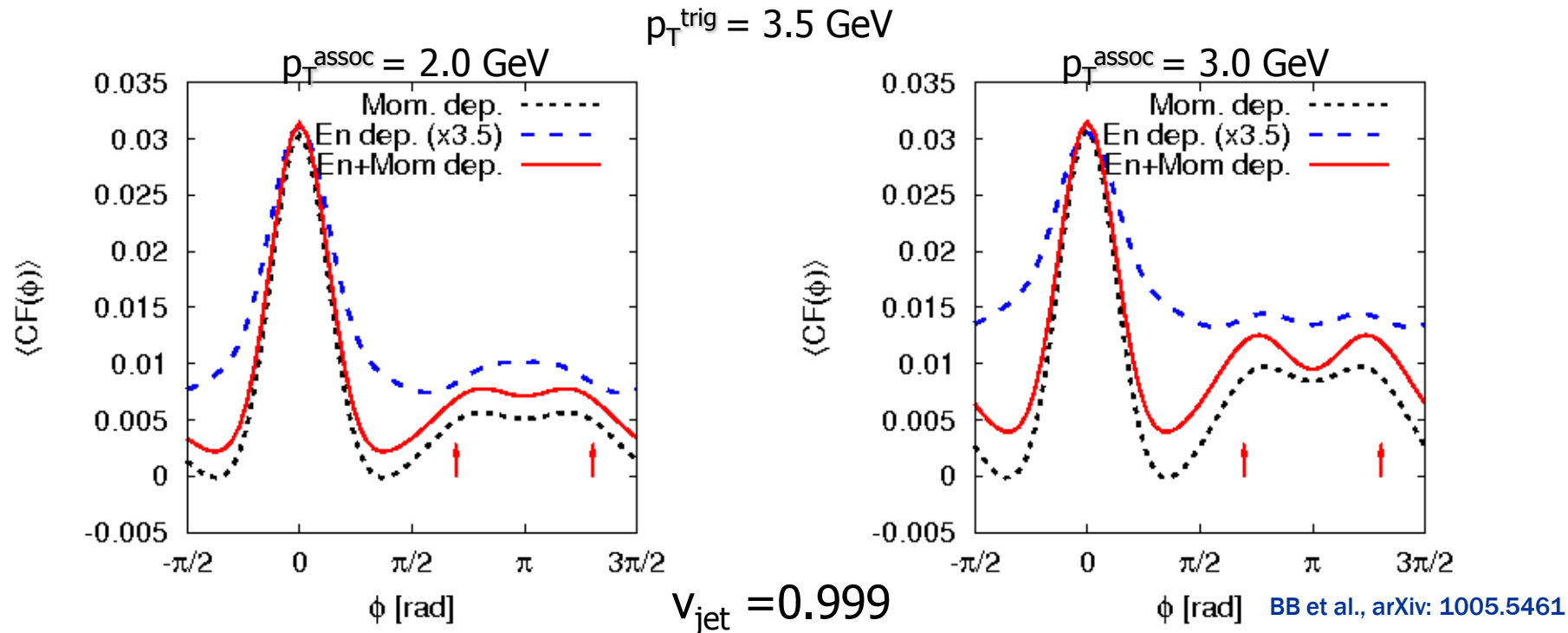
$v_{\text{jet}} = 0.999$



Expanding Medium IV

Comparing different deposition scenarios, one sees that

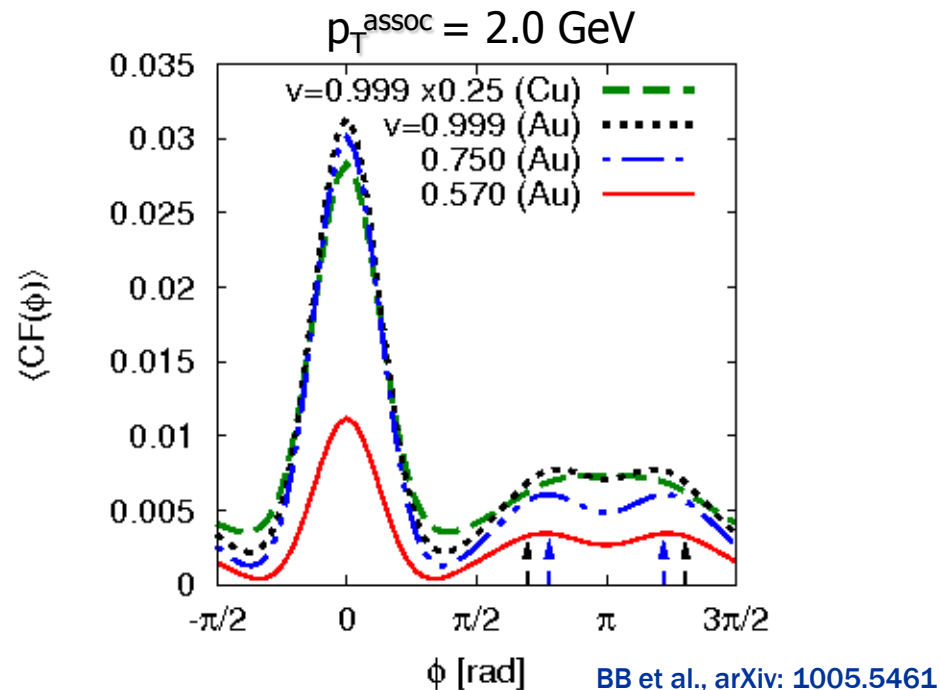
→ „cone“ angle approximately the same for different deposition scenarios



→ $p_T^{\text{assoc}} = 2.0 \text{ GeV}$: No double-peaked structure for pure energy deposition scenario due to thermal smearing

Expanding Medium V

Considering a bottom quark ($M=4.5$ GeV), propagating at $v_{\text{jet}} < c_s$
(on-shell energy-momentum deposition scenario)



- Conical emission angle also appears for **subsonic** jets ⇒ Not a Mach cone
- Cu+Cu: Similar away-side shoulder width,
double-peak structure reappears for $p_T^{\text{assoc}} = 3$ GeV

Summary

- Investigation of jet-medium interactions using (3+1)d ideal hydrodynamics for different energy and momentum loss scenarios (schematic source term, pQCD, AdS/CFT)
- Diffusion wake is always created if $dM/dx > \text{threshold}$
- Different impacts of pQCD and AdS/CFT source terms
- „Conical“ signal can be created:
 - by averaging over wakes created by jets in different events.
 - There is a deflection of particles emitted due to collective transverse flow.
 - Quite insensitive to deposition mechanism, jet velocity (even for *subsonic* jets), and system size
- ⇒ Structure unrelated to EoS
- ⇒ Can be tested experimentally comparing hard-soft correlations induced by heavy-flavor tagged jets.

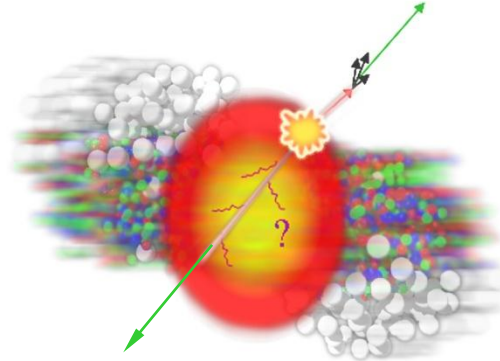
Backup

Punch – Through Jet I

Applying a static medium and an ideal Gas EoS for massless gluons

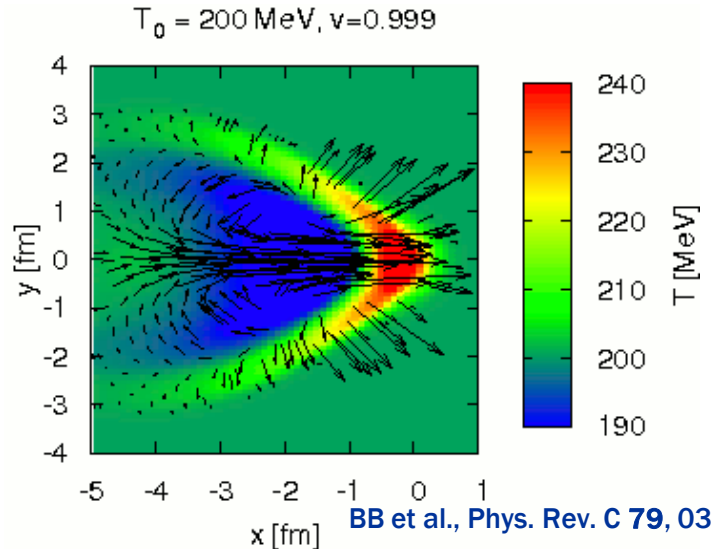
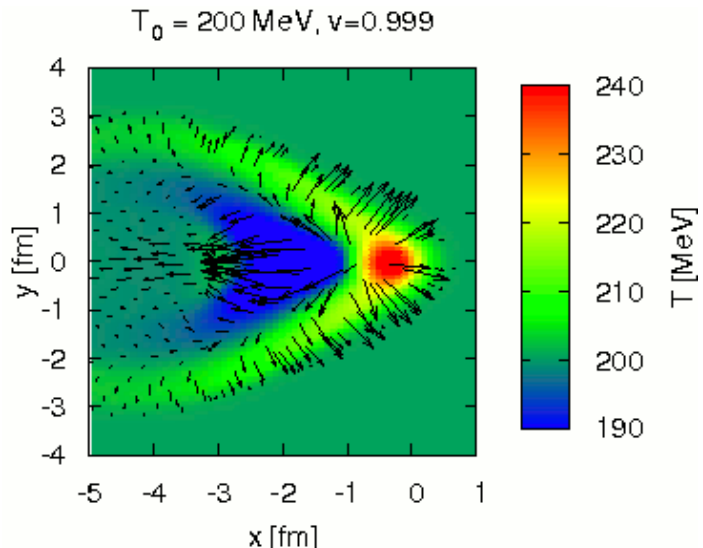
↳ Maximal fluid response

Assume: Near-side jet is not modified by medium



$$S^\nu = \int_{\tau_i}^{\tau_f} \frac{dM^\nu}{d\tau} \delta^{(4)} \left[x^\mu - x_{\text{jet}}^\mu(\tau) \right] d\tau$$

$$dM^\nu/d\tau = (dE/d\tau, dM/d\tau) \quad x_{\text{jet}}^\mu = x_0^\mu + u_{\text{jet}}^\mu \tau$$



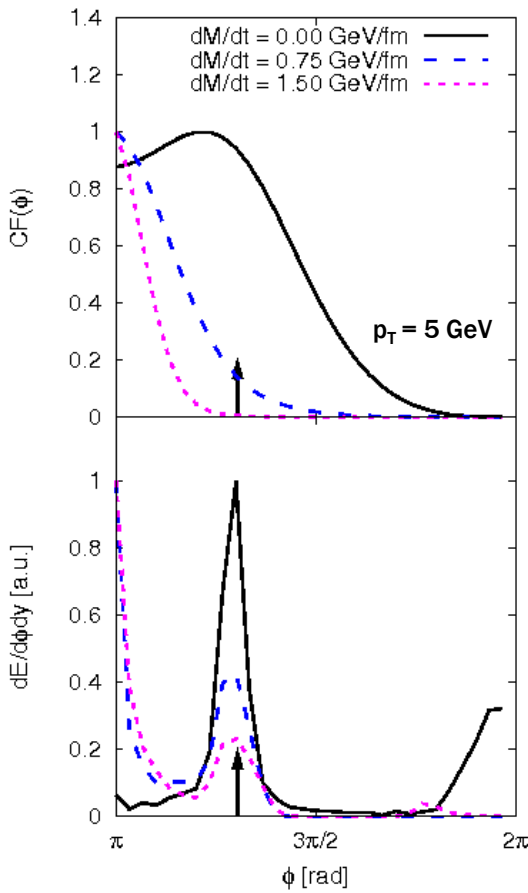
BB et al., Phys. Rev. C 79, 034902 (2009)

$$\frac{dE}{dt} = 1.5 \frac{\text{GeV}}{\text{fm}}, \quad \frac{dM}{dt} = 0 \frac{\text{GeV}}{\text{fm}} \quad v=0.999$$

$$\frac{dM}{dt} = \frac{dE}{dt} = 1.5 \frac{\text{GeV}}{\text{fm}} \quad t=4.5/v \text{ fm}$$

Punch – Through Jet II

BB et al., Phys. Rev. C 79, 034902 (2009)



Normalized, background-subtracted isochronous Cooper-Frye at mid-rapidity

$$\frac{dN}{p_T dp_T dy d\phi} \Big|_{y=0} = \int_{\Sigma} d\Sigma_{\mu} p^{\mu} [f_{\text{Boltzmann}}(u \cdot p/T) - f_{\text{eq}}]$$

Energy Flow Distribution

$$\frac{dE}{d\phi dy} \Big|_{y=0} = \int d^3 \vec{x} E(\vec{x}) \delta[\phi - \Phi(\vec{x})]$$

$$\Phi(\vec{x}) = \tan^{-1} \left[\frac{M_y(\vec{x})}{M_x(\vec{x})} \right]$$

Diffusion wake causes peak in jet direction

Assuming: Particles in subvolume will be emitted into the same direction

Punch – Through Jet III

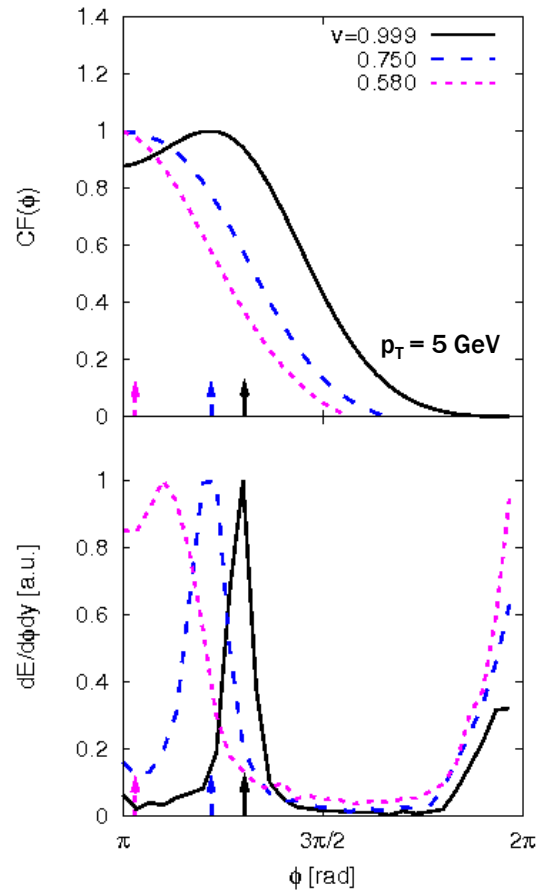
Does the jet-pattern reproduce the features of a Mach cone?

$$\cos \phi_M = c_s / v_{\text{jet}}$$

→ Velocity dependence of the emission angle

Creation of Bow Shock for smaller v strengthens peak in jet direction

BB et al., Phys. Rev. C 79, 034902 (2009)

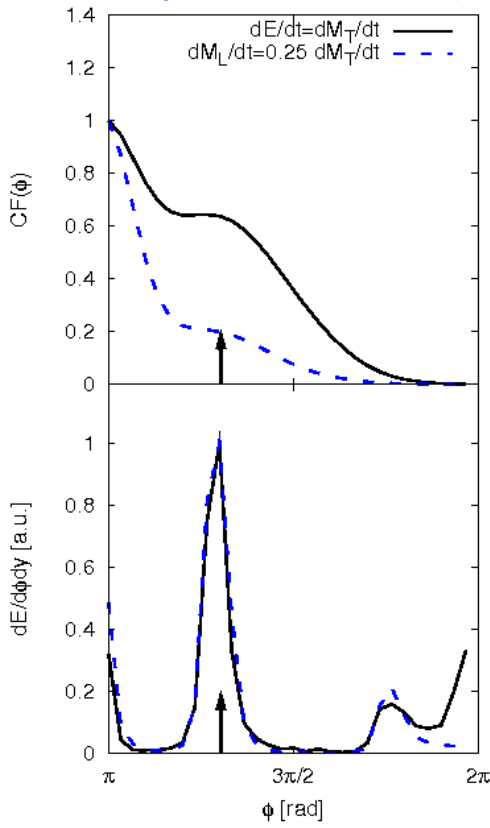


Punch – Through Jet IV

- Transverse momentum deposition:

$$S^\nu = \frac{1}{(\sqrt{2\pi}\sigma)^3} \exp\left[-\frac{(\vec{r}-\vec{x})^2}{2\sigma^2}\right] \left(\frac{dE}{dt}, \frac{dM_L}{dt}, \frac{dM_T}{dt} \cos\theta, \frac{dM_T}{dt} \sin\theta\right)$$

BB et al., Phys. Rev. C 79, 034902 (2009)

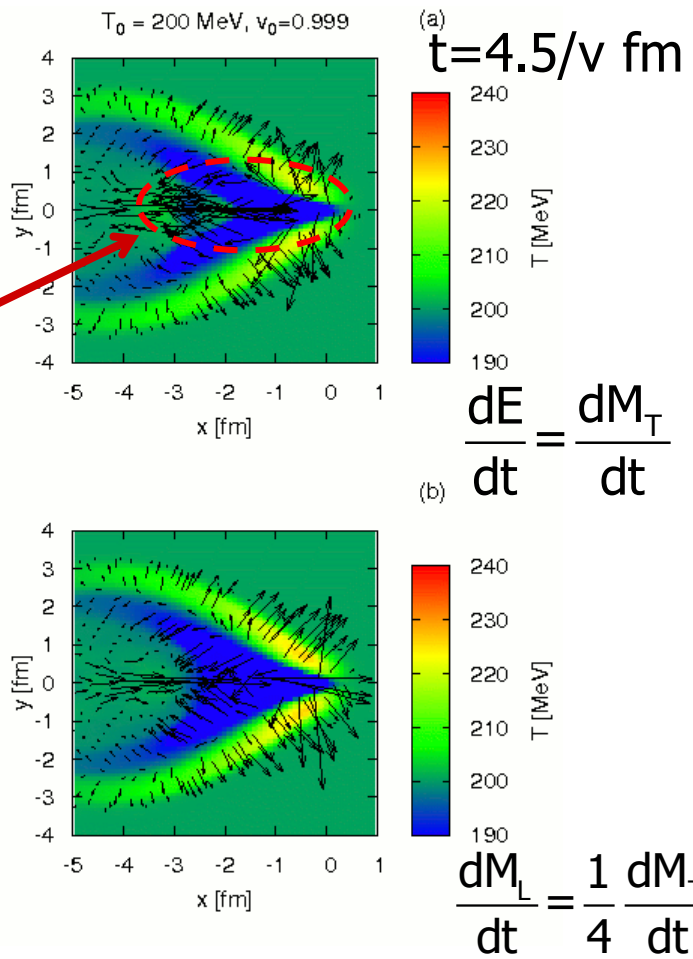


→ Still influence of diffusion wake



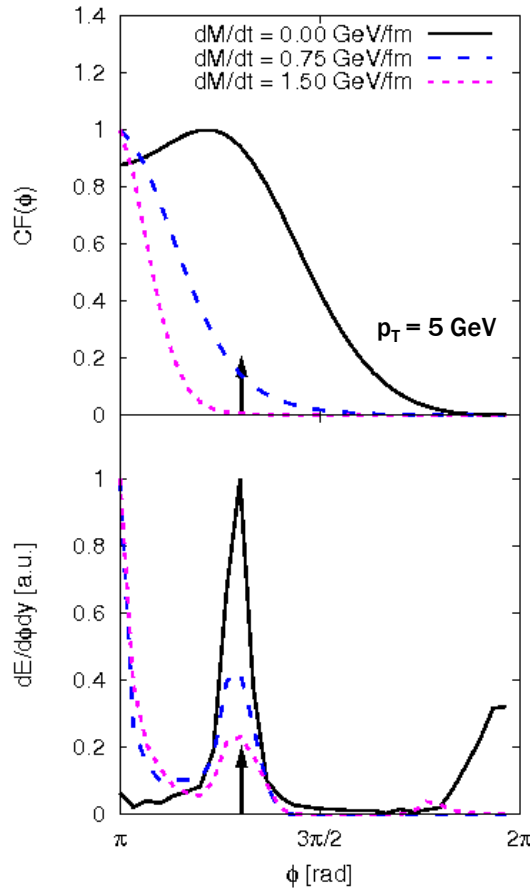
from explosion of matter

→ Vorticity conservation

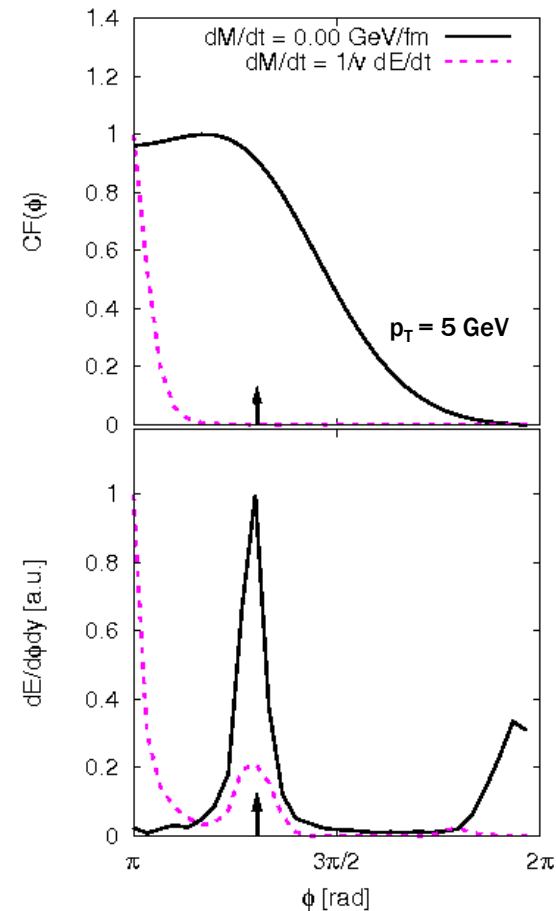


Punch – Through vs Stopped

BB et al., Phys. Rev. C 79, 034902 (2009)



Punch-Through Jet

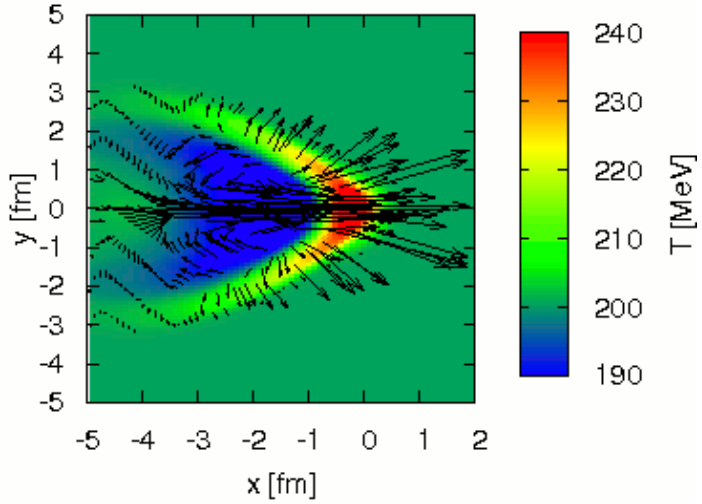


Stopped Jet

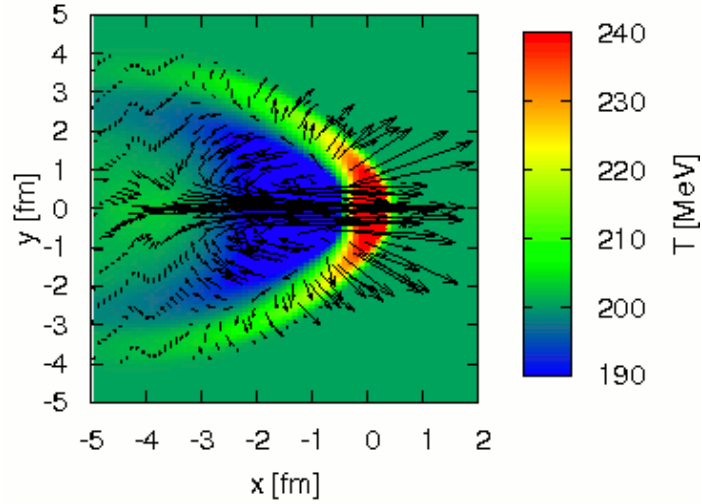
→ Similar freeze-out patterns

Punch – Through Jet: Velocity Scan

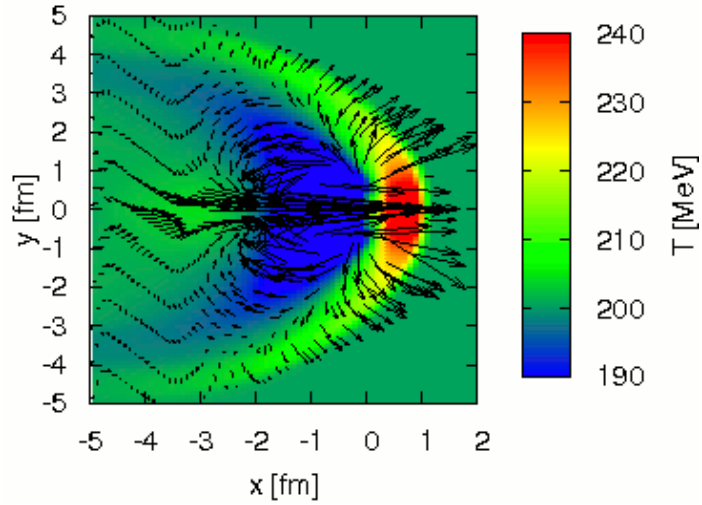
$T_0 = 200 \text{ MeV}, v=0.999$



$T_0 = 200 \text{ MeV}, v=0.75$



$T_0 = 200 \text{ MeV}, v=0.58$



$t=4.5/v \text{ fm}$

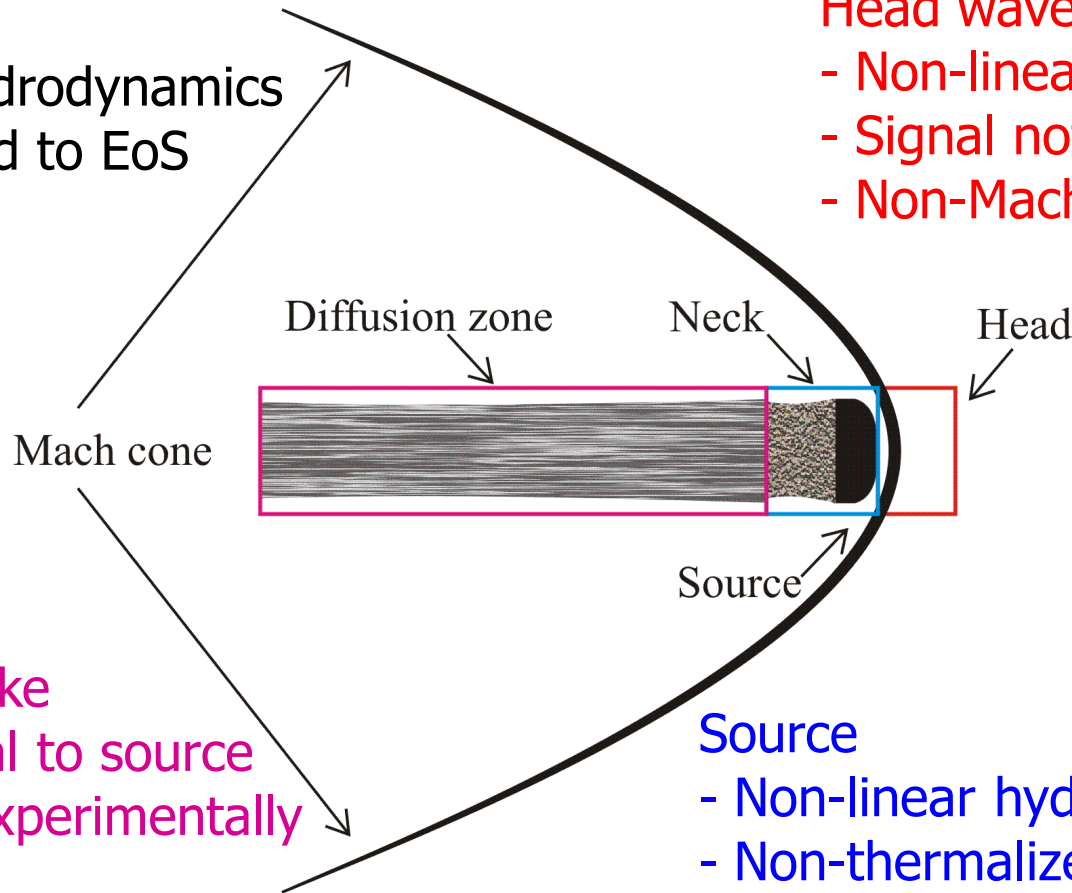
Why linearized Hydro is not so good

Mach Cone

- Linear hydrodynamics
- Connected to EoS

Head wave pile-up

- Non-linear hydrodynamics
- Signal not well understood
- Non-Mach cone angle



Mach cone

Diffusion zone

Neck

Head

Source

Diffusion Wake

- Proportional to source
- Not seen experimentally

Source

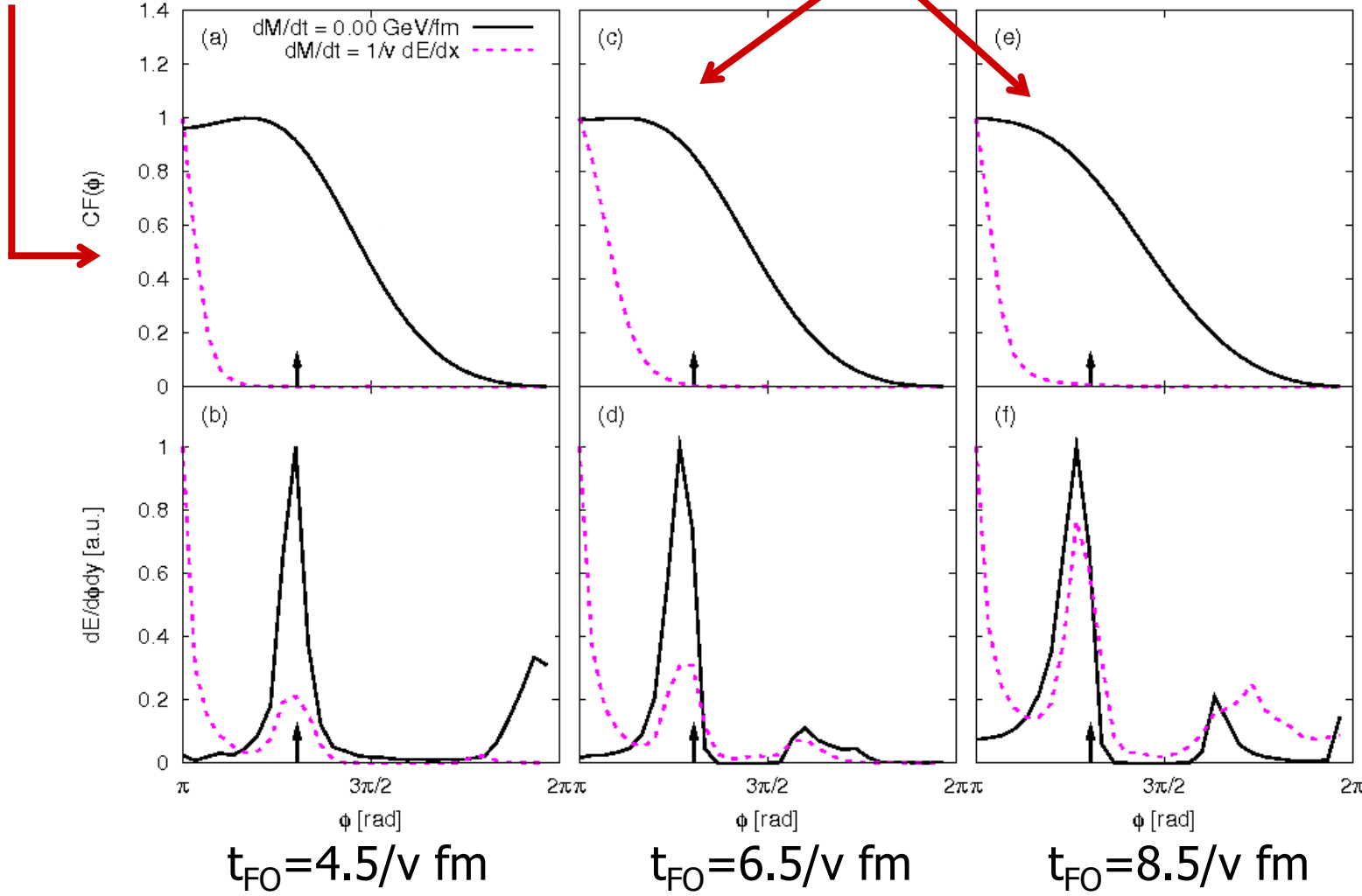
- Non-linear hydrodynamics
- Non-thermalized

Stopped Jet

Diffusion wake causes peak in jet direction

Larger impact of thermal smearing

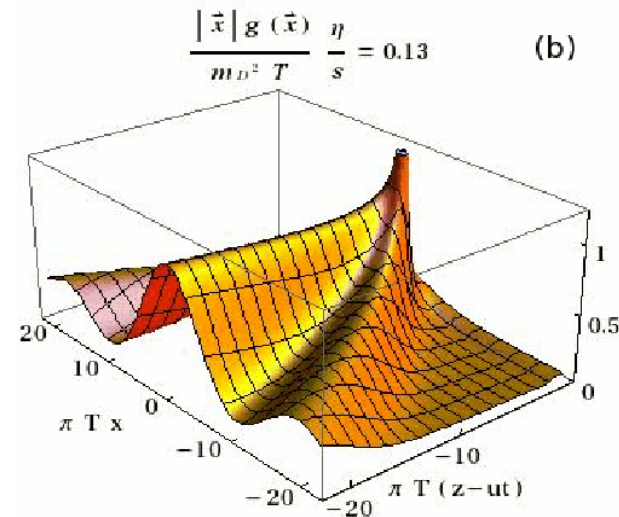
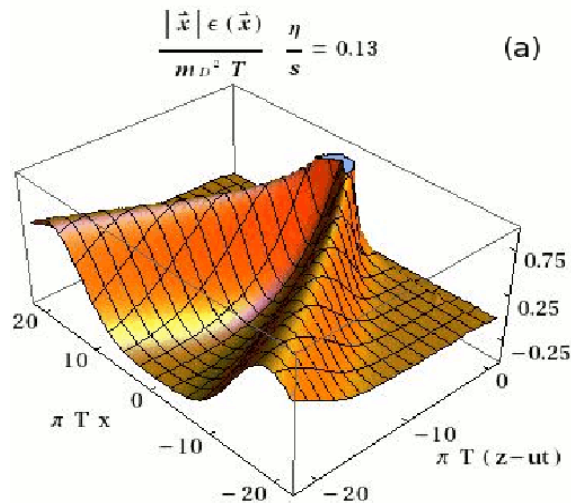
BB et al., Phys. Rev. C 79, 034902 (2009)



Jets in pQCD I

Considering a static medium and linearized hydrodynamics for a punch-through jet

R. Neufeld et al, Phys. Rev. C 78, 041901 (2008)

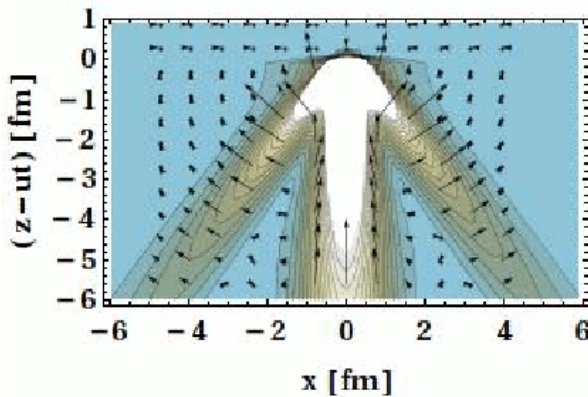


→ Mach cone signal & Diffusion Wake

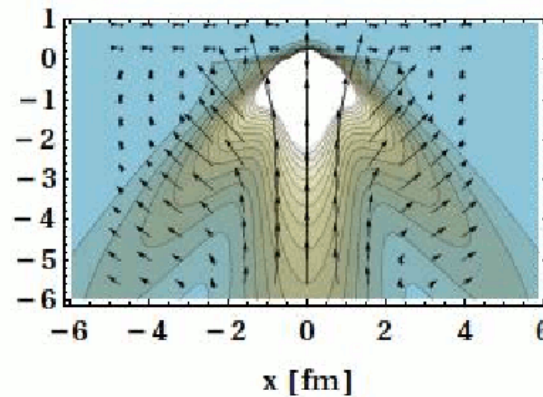
Jets in pQCD II

Contour plots of magnitude of perturbed momentum density

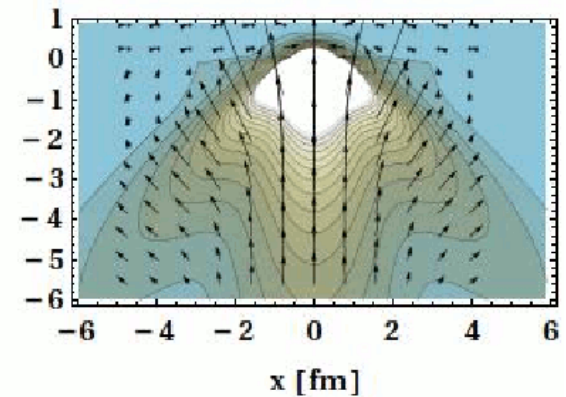
R. Neufeld et al., Phys. Rev. C 79, 054909 (2009)



$$\frac{\eta}{s} = \frac{1}{4\pi}$$



$$\frac{\eta}{s} = \frac{3}{4\pi}$$

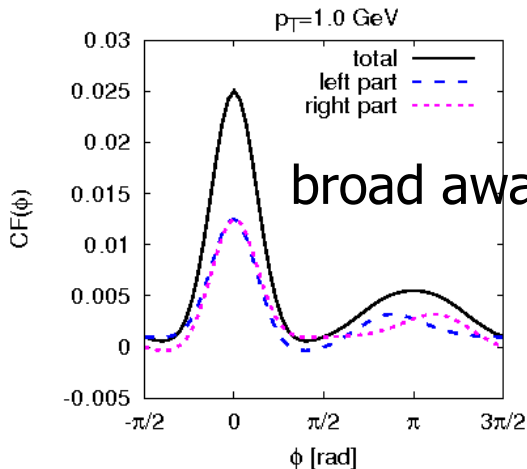


$$\frac{\eta}{s} = \frac{6}{4\pi}$$

→ Strong flow in jet-direction

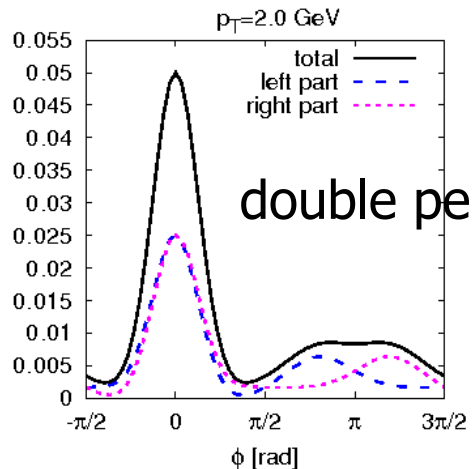
→ $\frac{dM}{dt}(t) > \frac{dE}{dt}(t)$

Expanding Medium

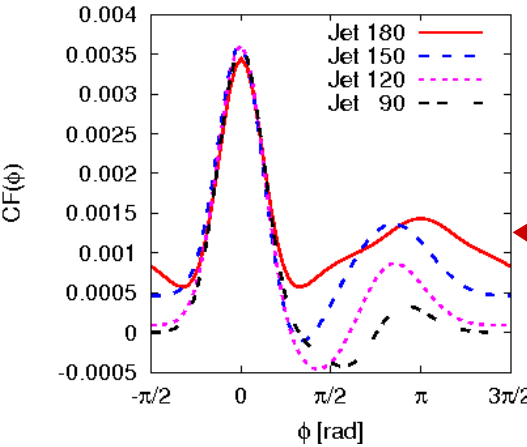


$E_{\text{tot}} = 10 \text{ GeV}$
 $p_{\text{T}}^{\text{trig}} = 7.5 \text{ GeV}$

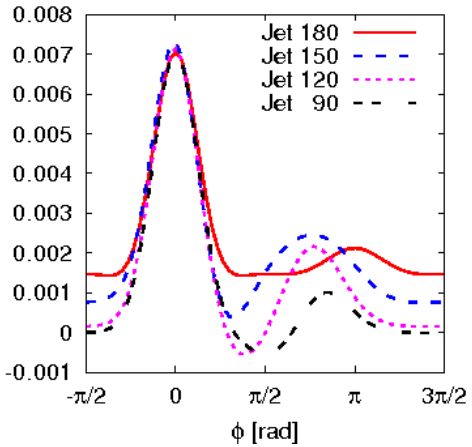
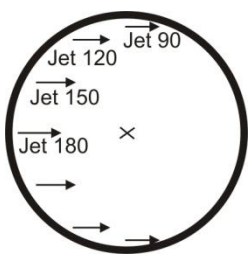
broad away-side peak



double peaked structure

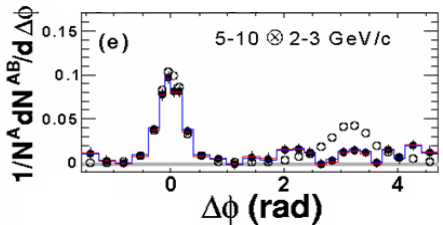


due to non-central jets



Strong impact of the Diffusion wake

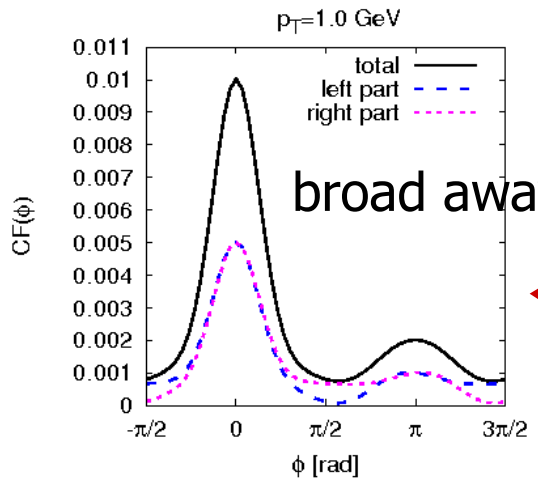
→ Causes smaller dip for $p_{\text{T}}=2 \text{ GeV}$



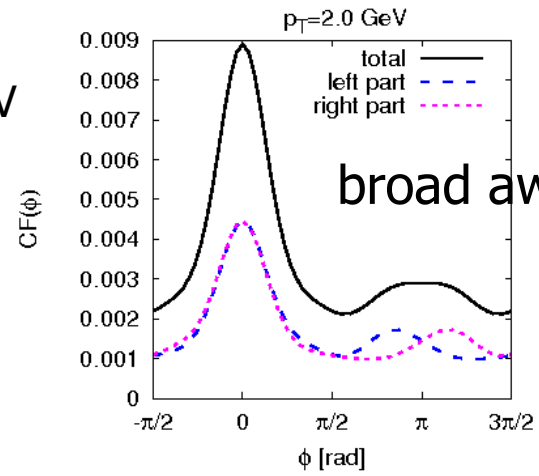
PHENIX, Phys. Rev. C 77, 011901 (2008)

Expanding Medium

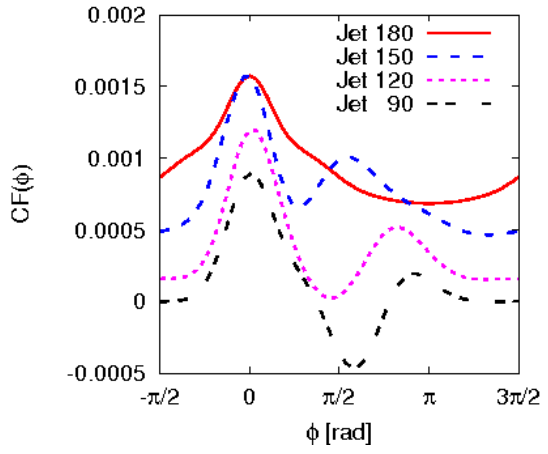
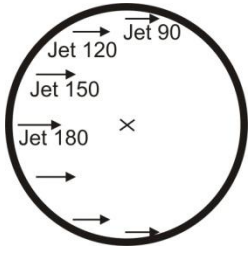
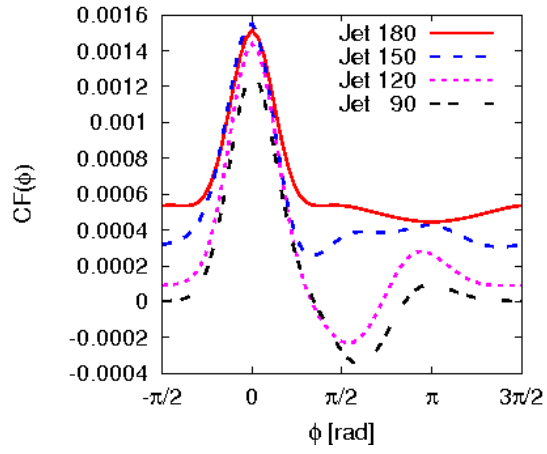
$E_{\text{tot}} = 5 \text{ GeV}$
 $p_{\text{T}}^{\text{trig}} = 3.5 \text{ GeV}$



broad away-side peak



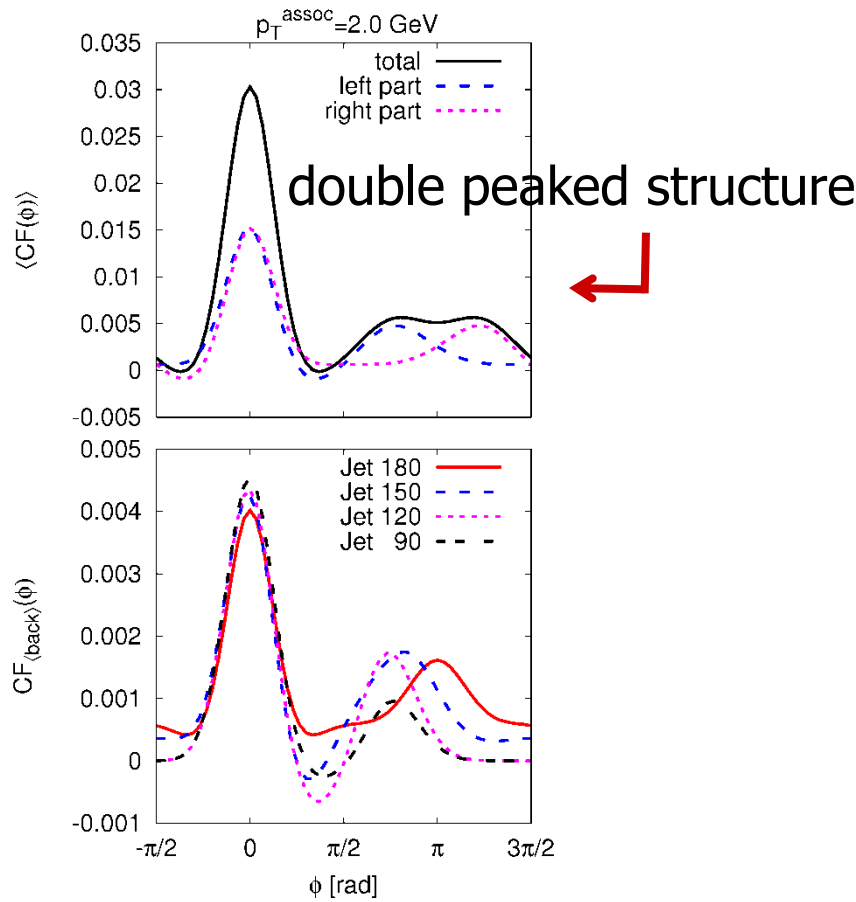
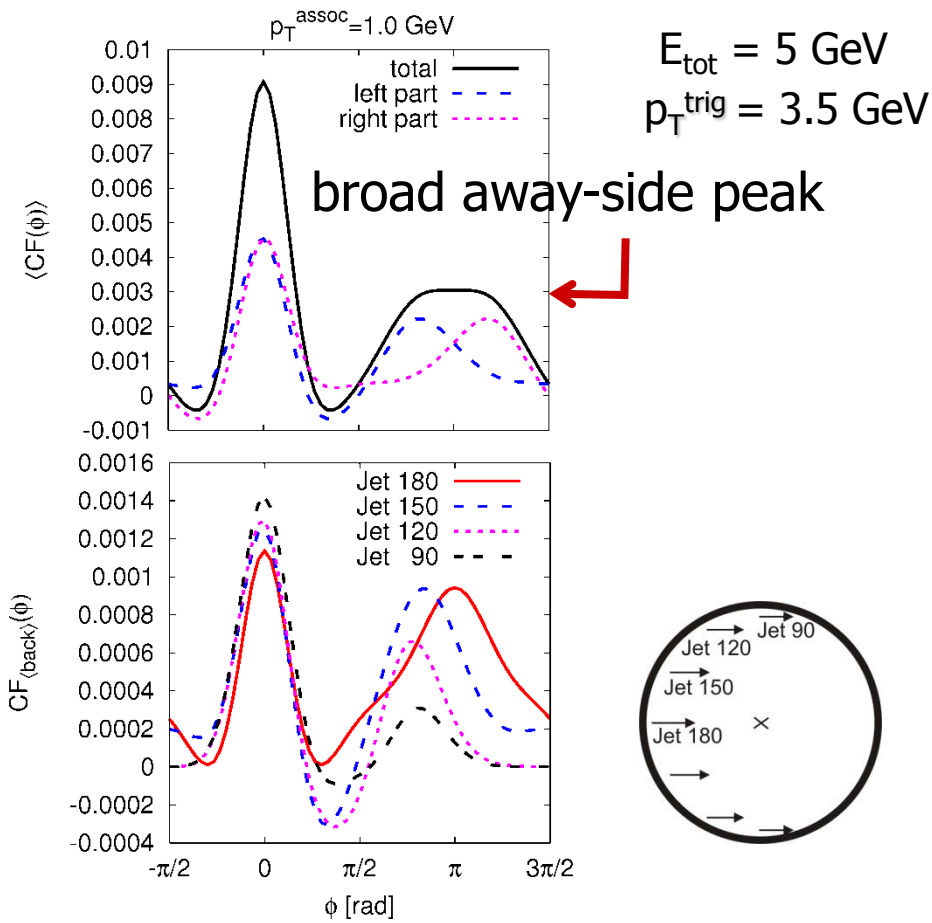
broad away-side peak



Pure energy deposition

- No conical distribution in expanding medium for $p_{\text{T}}=1 \text{ GeV}$ and $p_{\text{T}}=2 \text{ GeV}$
- Jet 180: No peaks on away-side

Expanding Medium

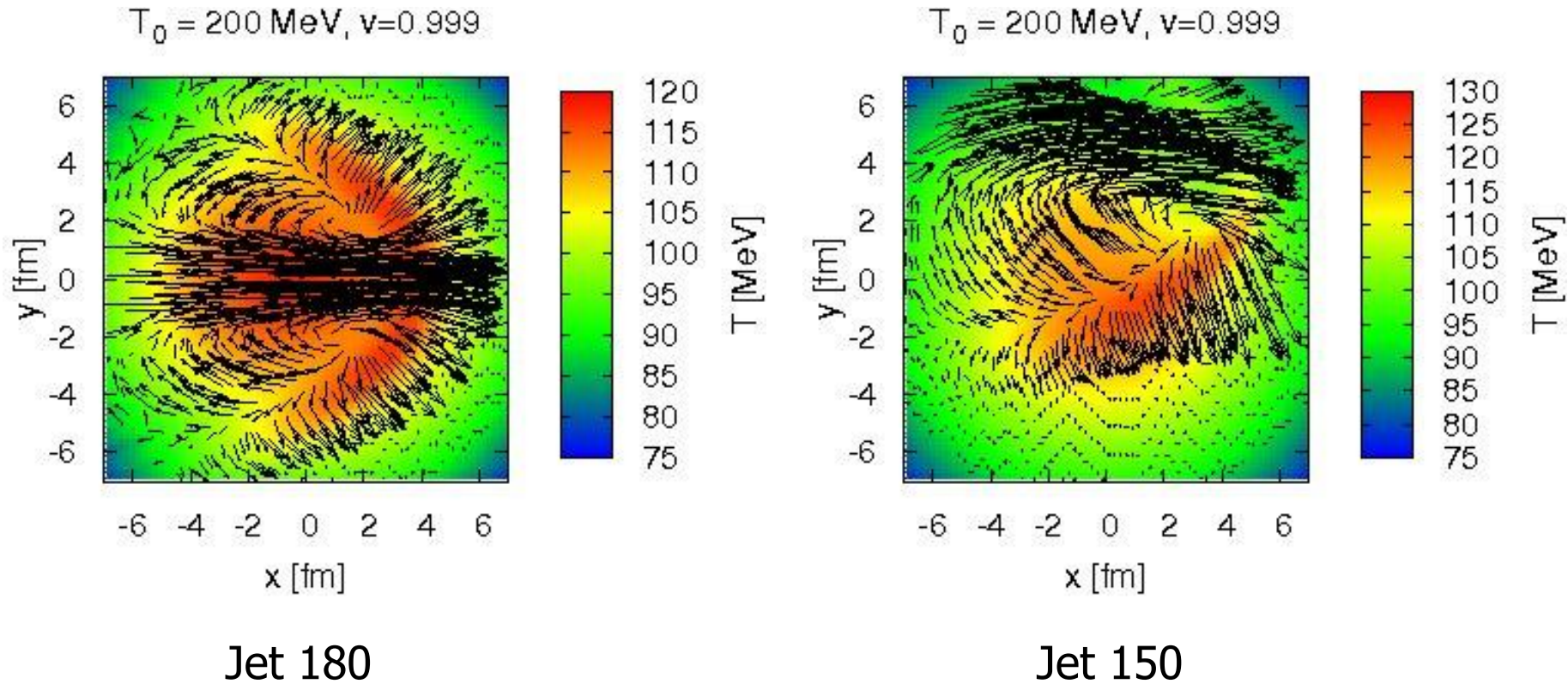


Pure momentum deposition

→ The same p_T -dependence as for energy and momentum deposition

Expanding Medium

Flow profile at freeze-out after background subtraction



The Caveat

- Assumption :
Correlations from flow anisotropy
and jets are uncorrelated

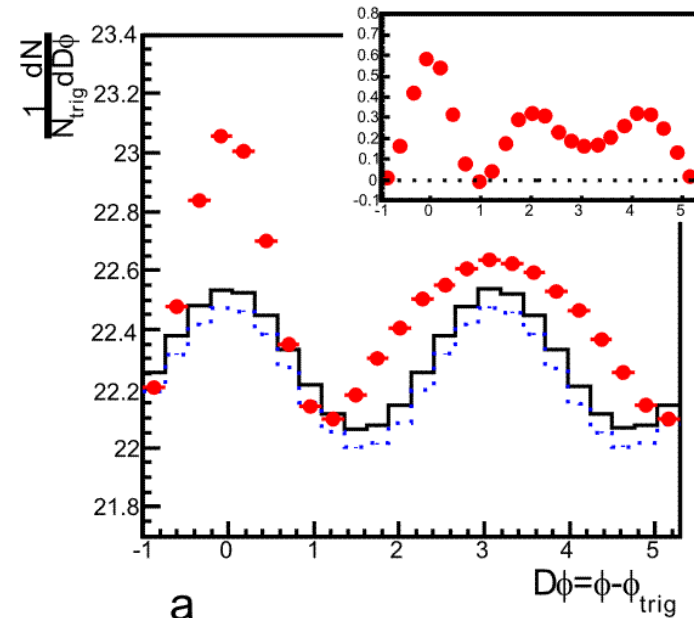
ZYAM (Zero Yield At Minimum)

- Subtraction of:
estimated elliptic flow modulated
background

- can leads to:
double peaked structure

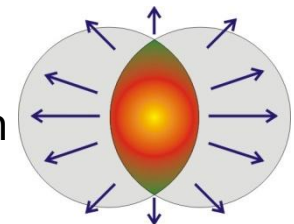
Two-source model:

$$C_2(\Delta\phi) = C_{2,\text{jet}}(\Delta\phi) + b [1 + 2\langle v_2^T \rangle \langle v_2^A \rangle \cos(2\Delta\phi)]$$



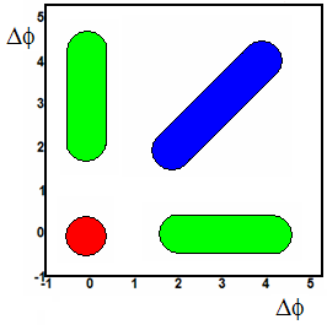
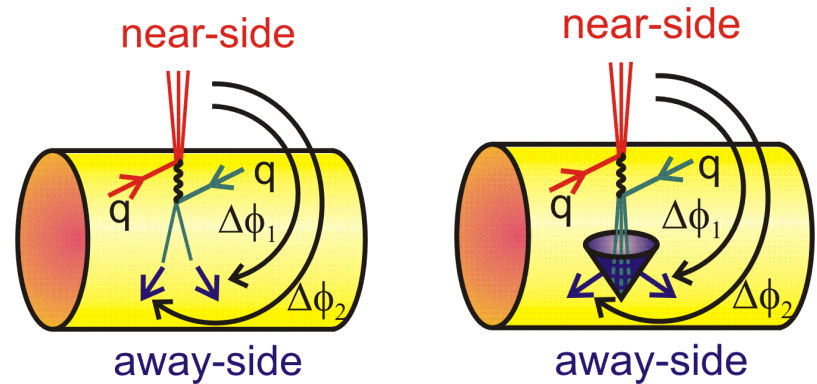
a
J. Ulery [STAR], PoS LHC07, 036 (2007)

Background:
Particle correlation
from elliptic flow

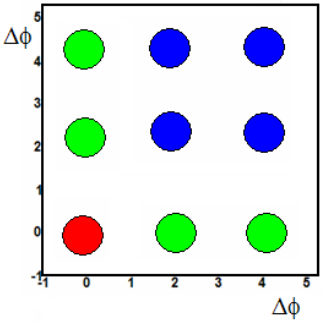


3-Particle Correlations

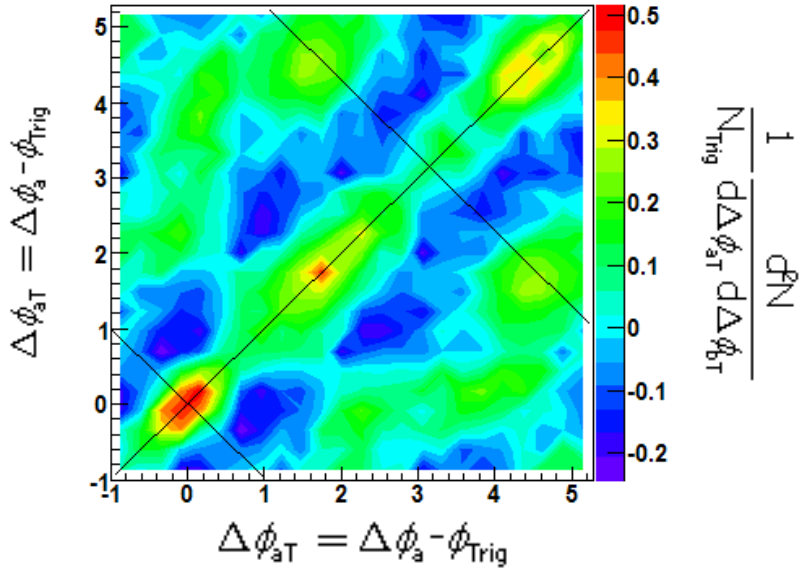
3-Particle Correlations seem to corroborate the Mach Cone picture



Deflected jet



Mach Cone

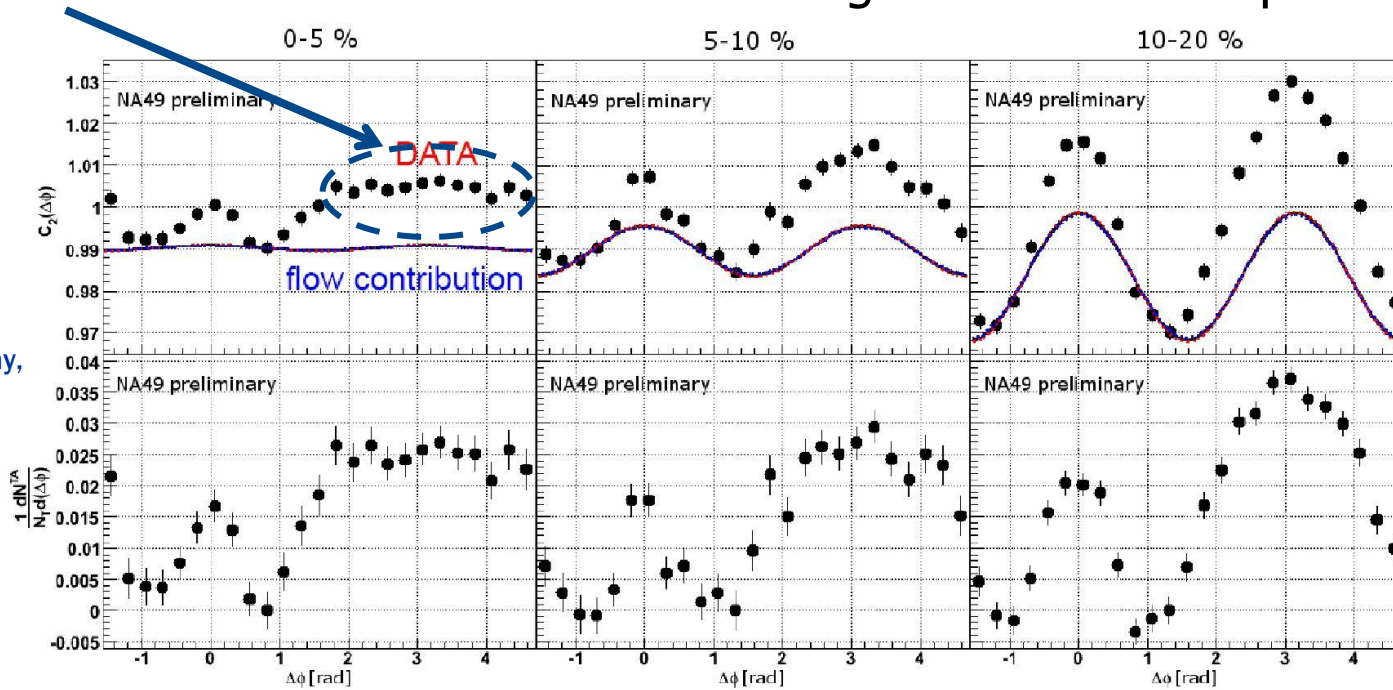


STAR Int. J. Mod. Phys. E 16 (2007)

→ Experimental data show 2 off-diagonal peaks

- High- p_T correlations at SPS energies show conical structure?

→ Conical structure even before subtracting the effect of elliptic flow - ZYAM

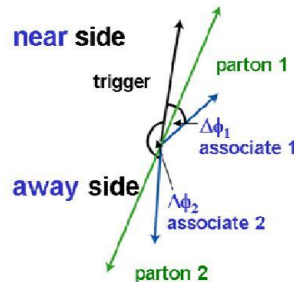


Pb+Pb at
158A GeV

Blume, Talk at
Confinement8,
Mainz, Germany,
2008

Correlation function:

$$C(\Delta\phi) = \frac{N_{same}^{pair}(\Delta\phi)}{N_{mixed}^{pair}(\Delta\phi)} \times \frac{\int N_{mixed}^{pair}}{\int N_{same}^{pair}}$$



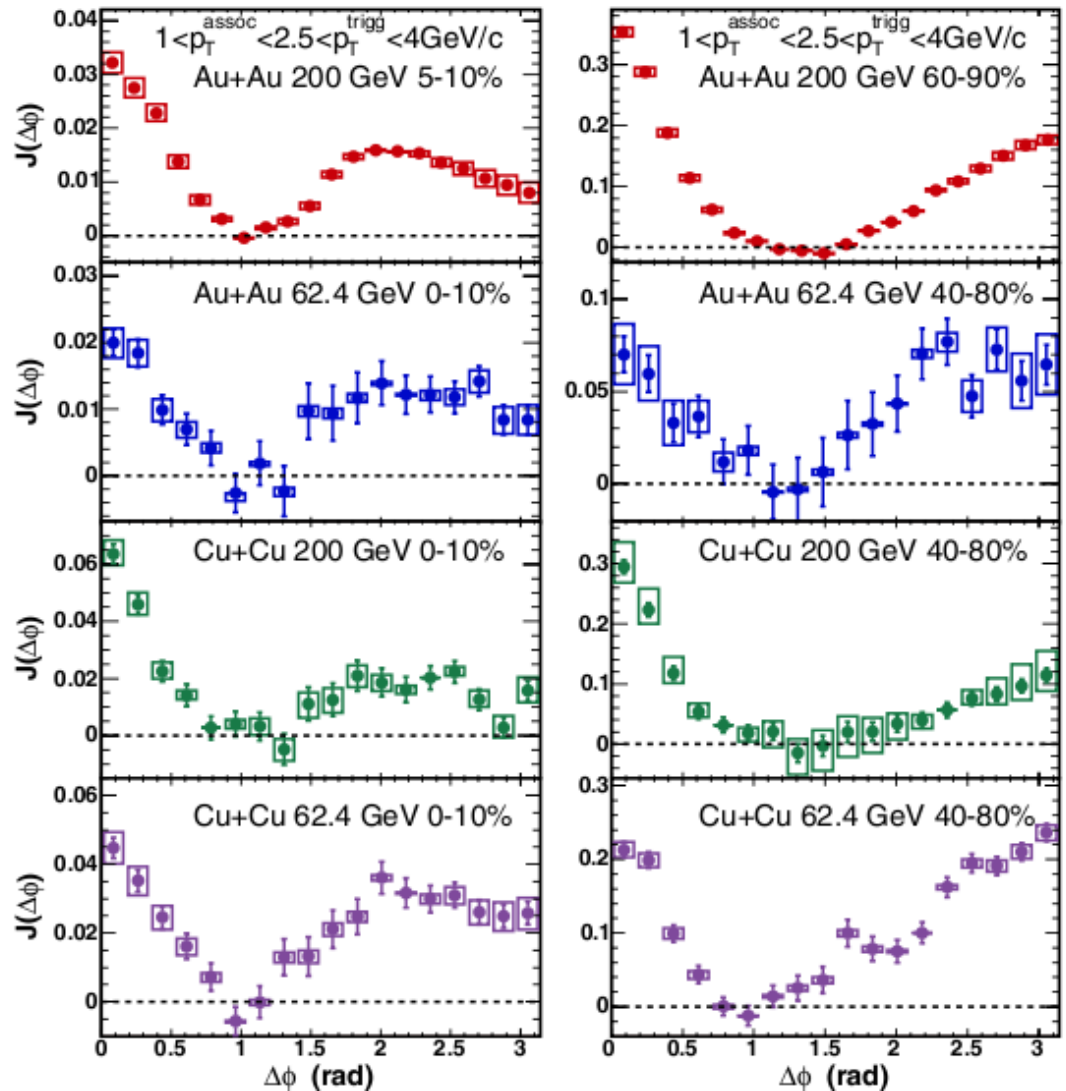
Conditional yield:

$$J_2(\Delta\phi) = \frac{1}{N^T} \frac{dN^{TA}}{d(\Delta\phi)} = \frac{C_2^{jet}(\Delta\phi)}{\int C_2(\Delta\phi') d(\Delta\phi')} \frac{N^{TA}}{N^T}$$

Flow contribution subtracted
using ZYAM method

Jet - Studies in HIC

For most central collisions, the shoulder width is similar in Au+Au and Cu+Cu collisions

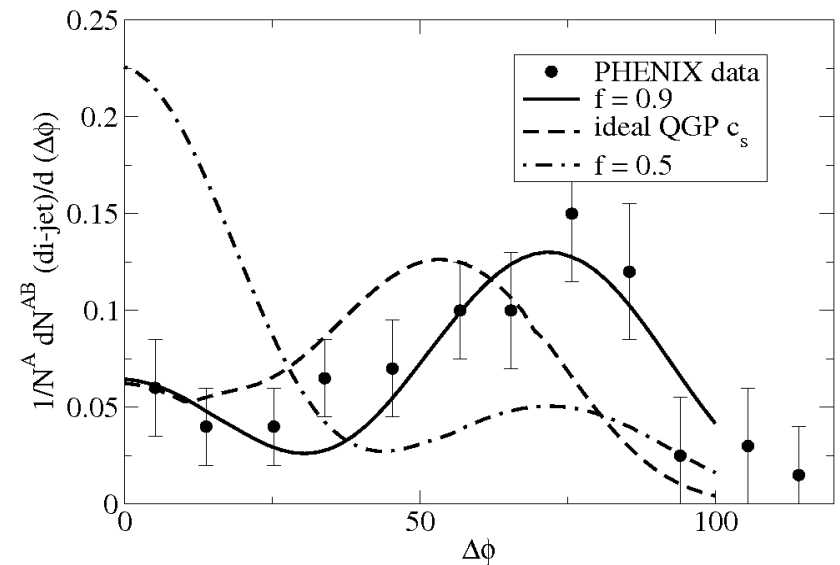


PHENIX, PRL98, 232302 (2007)

Jet-Medium Models

Determines angular correlation pattern

- Fireball model
- Lattice QCD EoS
- BDMPS-like energy loss



T. Renk and J. Ruppert, *Phys. Rev. C* **73**, 034907 (2006)

f : „fraction ... of energy lost to the medium [that] excites a collective mode“

→ sound wave

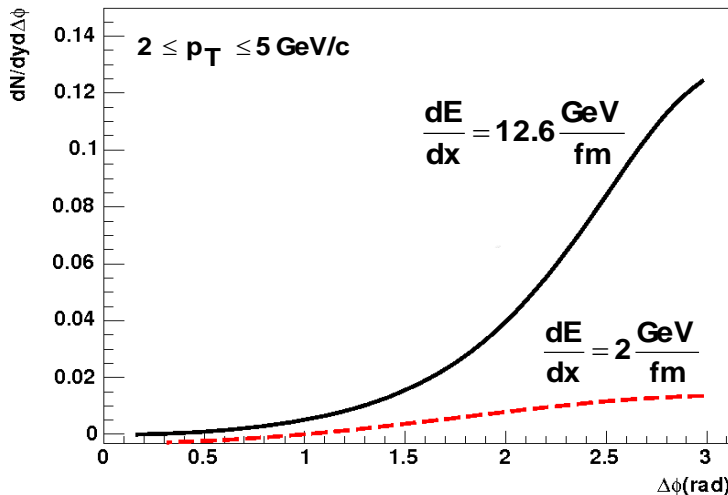
$(1-f)$: „remaining energy fraction ... [that] in essence heats the medium and leads to some amount of the collective drift along the jet axis...“

→ diffusion

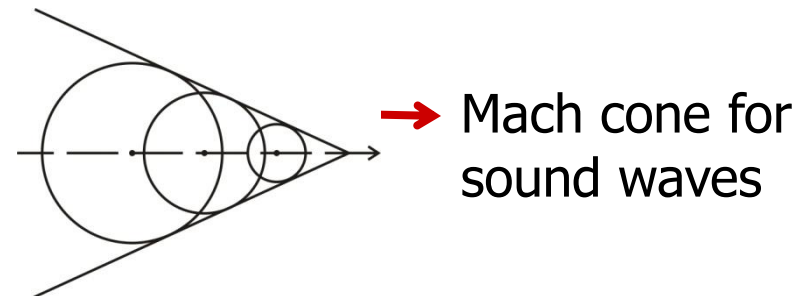
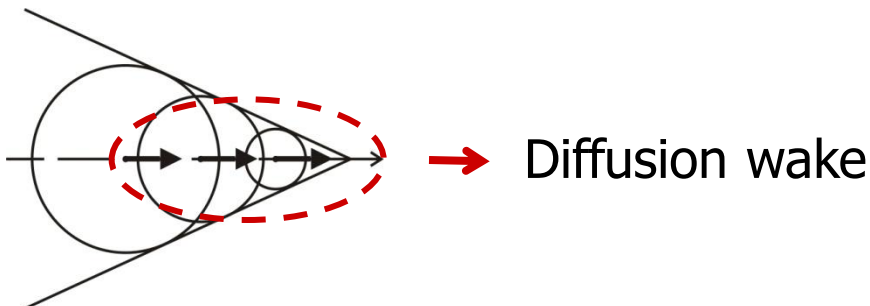
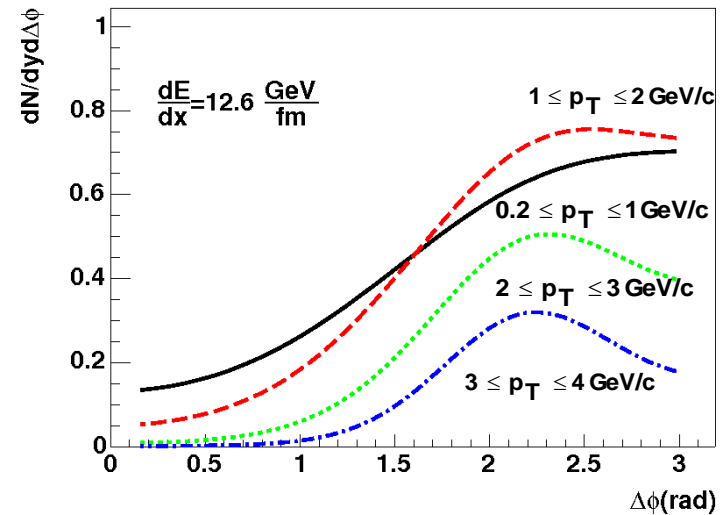
→ Mach cones **only** if $dM/dx \ll dE/dx$

Jet – Energy Loss Studies

- Jet deposits energy and momentum along a trajectory
- Applying linearized hydrodynamics

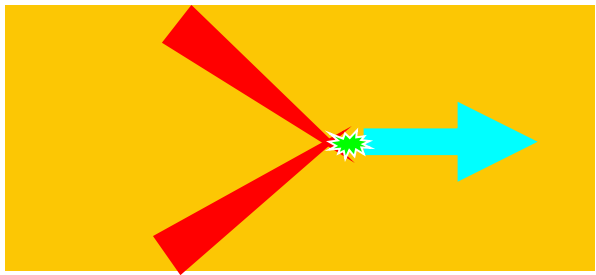


J. Casalderrey-Solana et al., Nucl. Phys. A 774, 577 (2006)

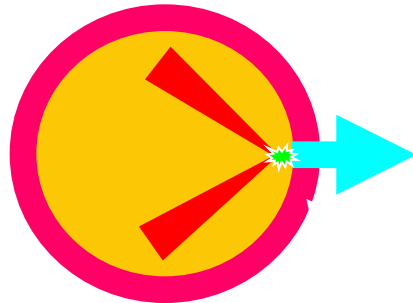
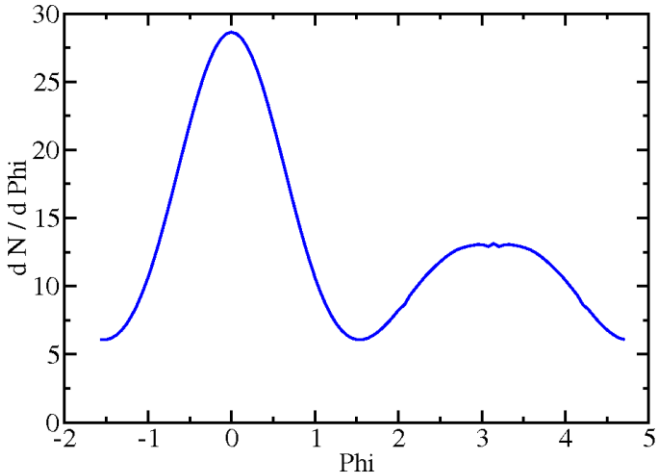


Jets in pQCD

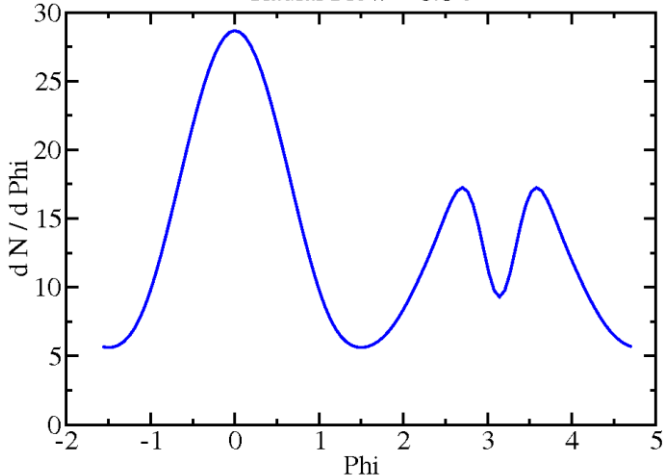
Comparing different freeze-out scenarios, **neglecting** the diffusion wake:



Cooper Frye Brick Freezeout Scenario



Cooper Frye Radial Freezeout Scenario
Radial Flow = 0.6 c



Neufeld et al, arXiv:0810.3185 [hep-ph]

→ Is the diffusion wake really an artefact???